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Technical Report

A Minimum Variance Parallel Interference Cancellation Receiver for DS-CDMA

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Abstract

This technical report presents a new modified multi-stage parallel interference cancellation receiver (MPIC) for a direct sequence code division multiple access system (DS-CDMA). The MPIC algorithm is based on the improved parallel interference cancellation (PIC) algorithm proposed by Divsalar *et al.* [1]. They proposed an iterative algorithm where the current decision estimate is a weighted combination of the current correlator output and the previous decision estimate. The concept of weighing the decision estimates is called partial PIC and the weighing coefficients are called partial parallel cancellation coefficients. In the improved PIC algorithm, the partial cancellation coefficients are independent of the number of users. In the proposed MPIC, the partial cancellation coefficients are calculated in order to minimize the variance of multiple access interference (MAI) after a single stage of interference cancellation. Simulation results show that a 4-stage MPIC receiver provides a significant capacity gain when compared to a conventional receiver for a given quality of service (QoS). The capacity gain is dependent on processing gain and $\frac{E_b}{N_0}$. For a bit error rate of 10^{-3}, a processing gain of 100, and $\frac{E_b}{N_0} = 10\text{dB}$, the capacity gain relative to a conventional receiver is in the order of 10. The complexity of MPIC is shown to be linear in the number of users. The MPIC algorithm assumes knowledge of phase and timing information and does not assume the knowledge of received signal powers. Unlike the earlier proposed receivers which assume knowledge of the received signal powers, the proposed MPIC algorithm estimates the received signal powers as part of the cancellation procedure.

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Chapter 1

Introduction

The emerging communications revolution poses many challenges to the designer of communications systems. One of the goals of the emerging standards is to provide communication to the user irrespective of location, whether mobile or fixed. The fact that there is an increasing need to support diverse services like voice, data and video in a seamless way to the end user underscores the importance of design of efficient communications systems. The ever increasing popularity of the cellular systems and the availability of limited spectrum lays greater emphasis on the design of spectrum efficient systems. Design of spectrum efficient systems can be achieved by efficient coding, modulation or by efficient multiple-access techniques.

CDMA is a multiple access technique which can support diverse services like voice, data and video and is being proposed as the multiple access technique for the third generation wireless systems. The choice of CDMA as the multiple access system for next generation wireless systems motivates us to investigate various methods of increasing the spectral efficiency of CDMA systems. Interestingly, for a CDMA system, the user capacity is not hard limited by bandwidth but is limited by the specified quality of service (QoS). The soft capacity limit of CDMA system is determined by a set of parameters like the required QoS, activity factor of the specified service and sectorization of the cell. In a CDMA system every user is allocated a unique code which modulates the narrow-band data signal to a wide-band signal. All the users communicate with the central station simultaneously and use all the bandwidth all the time.

In an asynchronous CDMA system, orthogonality between the different user codes cannot be maintained which results in multiple access interference (MAI). The conventional receiver's estimates are corrupted by MAI and the capacity of a CDMA system employing a conventional receiver is limited by MAI that can be admitted into the system to support a given QOS. A method to increase the capacity of a CDMA system is to reduce or cancel MAI. A number of interference cancellation receivers have been proposed in the literature. Two classes of algorithms which are built around the traditional conventional receiver are successive interference cancellation (SIC) and parallel interference cancellation (PIC) algorithms. This technical report aims at developing a new receiver algorithm based on SIC or PIC algorithm. Sub-optimal receivers based on SIC and PIC algorithms have already been proposed. The algorithms that have been proposed have been shown to support a larger user capacity than that supported by a conventional receiver. The central idea of this technical report is to develop a new algorithm for interference cancellation. As there are two contending algorithms it is essential that the algorithm that has a better performance is chosen for the development of a new algorithm. This thesis first addresses the performance of a SIC vis-a-vis a conventional receiver for a single-cell system as well as a multi-cell system, in order to answer the question of which receiver algorithm can support larger number of users. An information-theoretic approach is used to analyze the performance of a CDMA system employing a SIC receiver. The performance of a SIC receiver is compared with an optimal receiver for both the single-cell system and multi-cell system. The results suggest that a PIC scheme performs better than a SIC scheme and hence the PIC scheme is chosen as the algorithm on which the new receiver algorithm is developed.

A related but interesting problem of bandwidth assignment is investigated for the conventional receiver as well as a SIC. In a CDMA system the narrow-band data spectrum is spread to a wider bandwidth by modulating the data signal by a pseudo-noise (PN) code. Another way to spread the spectrum of the data signal is to apply error control coding. The resultant increase in the bandwidth of the narrow-band signal is defined as bandwidth expansion. For a CDMA system, bandwidth expansion could be due to modulation by PN codes or due to application of error control codes or a combination of both. The effective bandwidth assignment is one which maximizes the user capacity of the system.

Various multiple-access techniques are discussed in section 1.1 in order to present an introduction to the technical report topic. In section 1.2, the motivating factors for the use of multiuser receivers along with a short review of multiuser demodulation is presented. The aim and organization of this technical report is presented in the last section.

1.1 Multiple Access Techniques

Multiple access schemes allow multiple users to access a common communications channel. From a broad perspective, the two multiple access domains are space domain and time-frequency domain as time and frequency domains are duals of each other via Fourier transform. In the space domain multiple access scheme every user is separated from other users either by using spot beams as in the case of wireless communication or different cables as in wire-line communication. Cell sectorization in the existing cellular systems can be thought of as space division multiple access scheme.

In the time-frequency domain, frequency division multiple access (FDMA), time division multiple access (TDMA), and CDMA are the three major access techniques. Multiple access is facilitated by assigning users a set of mutually orthogonal signals. For a TDMA system, orthogonality is maintained by assigning different time periods for transmission. Hence, in TDMA orthogonality is maintained in time domain. In an FDMA system orthogonality is maintained by using mutually orthogonal frequencies for different users. For a CDMA system, the user codes are chosen from a mutually orthogonal set. An example of an orthogonal codes set is Walsh codes sequence. The number of users supported by any multiple access system is dependent on the dimensionality of the assigned space. Theoretically, all orthogonal multiple access scheme support the same number of users.

FDMA is the oldest multiple access technique and was used in the earliest cellular standards. FDMA assigns individual channels to individual users and the receiver can seperate the various users by employing simple bandpass filters. The number of channels determines the number of users that can be supported by this scheme. To support more number of users at the same data rate, the individual channel bandwidth should be reduced at the cost of increasing transmitting power.

In TDMA scheme, each user is assigned a time slot during which he transmits information. This scheme is applicable for digital modulation and requires synchronization of all the users transmissions. The number of users that can be supported depends on the number of time slots available, which is dependent on the data rate and the bandwidth of the system.

CDMA originated from spread-spectrum applications where anti-jamming and low probability of detection were the desired objectives. A review of spread-spectrum techniques can be found in papers by Scholtz [2] and Pickholtz *et al.* [3]. A number of books have also dealt with spread-spectrum techniques [4]-[8]. In a spread-spectrum system, a narrow-band data signal is modulated by a wide-band signal which in most cases is a pseudo-random signal. There are many methods in which CDMA can be implemented like frequency hopping (FH), multi carrier (MC), direct sequence (DS) and time hopping (TH). Direct sequence code division multiple access (DS-CDMA) has been proposed as the multiple access technique for the current IS-95 standard. In DS-CDMA system the narrow-band data signal is multiplied by a wide band signature sequence/user code thus expanding the narrow-band signal to a wide-band signal. Hence in DS-CDMA system users are neither separated in time-domain or frequency domain but are separated in a code domain, and all the users signals occupy the complete bandwidth. The data of users can be separated at the receiver since the signature sequences are unique to each user.

FDMA and TDMA are orthogonal in nature, whereas the DS-CDMA system can be designed to be either orthogonal or non-orthogonal system. In an asynchronous CDMA system, it is difficult to design an orthogonal signature sequence set. Hence an asynchronous DS-CDMA system is in general a non-orthogonal system. In general, a non-orthogonal system can support more number of users since there is no finite dimensionality of the signal space as there is in orthogonal systems.

The question as to which multiple access technique provides maximum capacity has been studied extensively. Information theory states that the maximum channel capacity is obtained by letting all the users to use all the bandwidth all the time [9]. CDMA, in fact, is the multiple access scheme which allows all the users to use all bandwidth all the time. Hence, from an information theory view, a CDMA system supports more capacity.

CDMA has been implemented in cellular systems and the current standard is IS-95 standard, a second generation wireless cellular system and has a relatively narrow channel bandwidth of 1.25 MHz. There are several third generation DS-CDMA standards which are under development and some of the standards propose the use of a multiuser detector.

1.2 Multiuser Demodulation

As stated earlier, in general, a non-orthogonal system can support more number of users than orthogonal schemes. The cost of designing a non-orthogonal multiple access scheme is that the conventional matched filter receiver is not the optimal receiver. The optimal demodulator for a multi-user system is a receiver which does a joint decoding of all the users [9]. As an asynchronous CDMA system is non-orthogonal in nature it can theoretically support more number of users. A multi-user demodulator which does joint decoding of the received signal has to be employed to realize the benefit of non-orthogonality in the system. Since, a conventional receiver is not the optimal receiver for a multi-user system the full potential of the system is not realized. This necessitates the need to employ an implementable receiver which performs a joint decoding of all the users.

An additional problem when a cellular system is considered is the near-far effect. Users near the base station are received at a higher power level than users near the boundary of the cell. The user with the highest received power swamps the signals with less received signal power. Hence, the multi-user detector that is to be employed should perform in the near-far environment also. In the current IS-95 cellular standard, strict power control is maintained so that every user is received at the same power level.

Schneider [11] was the first to study the multiuser detector and he presented a zero forcing de-correlating detector. Later Kashihara [12] and Kohono *et al.* [13] studied multiuser interference cancellation receivers. Verdu [14] presented the maximum likelihood multiuser detector which consists of a bank of matched filters followed by Viterbi algorithm, and he also showed that the CDMA systems are neither interference nor near-far limited but they are the limitations of the conventional receiver. The optimal algorithm has a complexity which is exponential in the number of users and hence is difficult to implement. Many sub-optimal detectors have been proposed and a review of these detectors is presented in chapter 2, and reviews on sub-optimal detectors can be found in [15, 17, 18].

1.3 Motivation and Organization of technical report

The aim of this technical report is to develop a multiuser interference cancellation receiver. The proposed receiver should have a complexity which is linear in the number of users. The technical report can be broadly divided into two sections. In the first section, the problem of the choice of which IC scheme should be the basis for developing a new algorithm is analyzed. To answer this question an information theoretic approach is developed to evaluate the performance of a receiver. Based on the results obtained from first section, a new algorithm is developed in the second section. The second section of the technical report presents the IC scheme that is modified and the new proposed algorithm.

Chapters 2 and 3 address the problem of choice of IC scheme. The proposed algorithm is developed in chapters 4 and 5. Conclusions and future directions are presented in the last chapter.

In chapter 2, a generic CDMA system model is presented which is used to analyze the capacity of a conventional receiver. The performance analysis of a conventional receiver is followed by a review of the work in the area of multiuser detection. Performance of a SIC receiver is analyzed in chapter 3. The analysis is performed for a single-cell system as well as a multi-cell system. The capacity gain when a SIC receiver is employed in the system is compared to the gain of an optimal interference cancellation receiver. The approach used to evaluate the capacities of IC and conventional receiver is discussed in chapter 3. This chapter shows that for a geometric user power distribution a SIC receiver provides a capacity gain which is significantly less than an ideal interference cancellation receiver. Hence PIC scheme becomes the basis for developing a modified interference cancellation receiver. The concept of parallel IC and already proposed parallel interference cancellation receivers is discussed in chapter 4. Chapter 5, introduces the proposed modifications to the improved PIC algorithm. Simulation results of the MPIC are also presented in this chapter. The complexity and implementation issues of a MPIC receiver are also discussed in chapter 5.

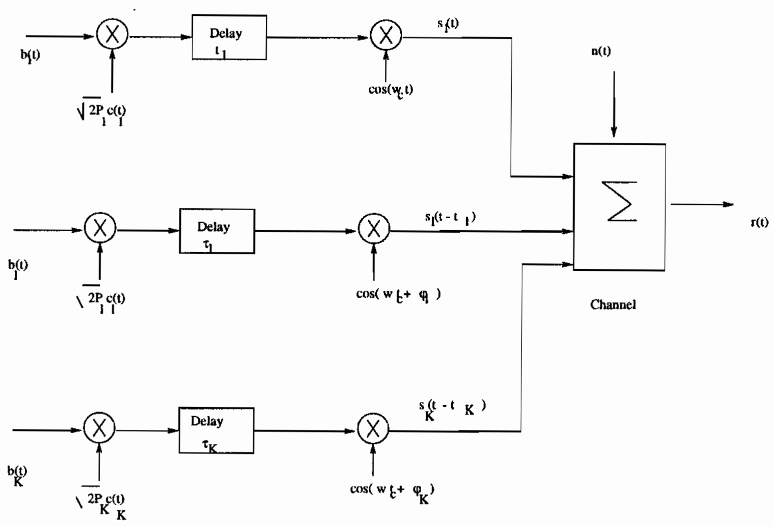
Chapter 2

Performance Analysis of a Conventional Receiver

In this chapter, the performance of a conventional receiver is analyzed. The analysis of a conventional receiver is presented in order to understand the effect of MAI on the capacity of the system and also to highlight the necessity for employing a multi-user detector. The system model used in this chapter and subsequent chapters is outlined in section 2.1. Performance of a conventional receiver is presented in section 2.2. The related work in the area of multiuser demodulation and some of the proposed multiuser detectors are presented in section 2.3.

2.1 DS-CDMA Single Cell System Model

The system model is shown in Fig. 2.1. It is assumed that there are K users in the system transmitting information through a common communication medium to a central receiver, which in case of a cellular system is the base station. All the users are assumed to be at the same data rate and communicating with the central station in an uncoordinated manner. Each user's signal passes through a distinct channel and is received at the base station. A Gaussian noise process is added to the aggregate outputs of the channel. The signal received at the centralized receiver from the k^{th} user is given by



Asynchronous DS-CDMA System Model

Figure 2.1: Asynchronous CDMA System Model

[19, 20]

$$s_k(t-\tau_k) \quad = \quad \sqrt{2P_k}c_k(t-\tau_k)b_k(t-\tau_k)\cos(w_c t+\phi_k) \tag{2.1}$$

where $b_k(t)$ is the data sequence for user k, τ_k is the delay for user k, P_k is the received power of user k and ϕ_k is the carrier phase offset of user k. Since τ_k and ϕ_k are relative terms we will assign $\tau_0 = 0$ and $\phi_0 = 0$. Both $c_k(t)$ and $b_k(t)$ are binary sequences with values -1 and +1. The PN sequence $c_k(t)$ is of the form

$$c_k(t) \quad = \quad \sum_{j=-\infty}^{\infty}\sum_{i=0}^{M-1} c_{k,i}\pi\left(\frac{t-(i+jM)T_c}{T_c}\right) \tag{2.2}$$

where M is the PN sequence period. $\pi(t)$ represents the unit pulse function, and i is an index to denote a particular chip within a PN cycle.

$$\pi(t) \quad = \quad \begin{cases} 1 & 0 \le t \le 1 \\ 0 & otherwise \end{cases} \tag{2.3}$$

For the user data sequence $b_k(t)$, T_b is the bit period. The chip sequence $c_k(t)$'s period is equal to $T_c = \frac{T_b}{N}$, where N is the processing gain. The user data sequence is given by,

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_{k,j}\pi\left(\frac{t - jT_b}{T_b}\right) b_{k,j}\{-1, 1\} \tag{2.4}$$

In a cellular CDMA system users communicate with the base station simultaneously over a common medium. Hence the received signal is,

$$r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t) \tag{2.5}$$

where, n(t) is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$. A simple AWGN channel is considered throughout this technical report. As can be observed from Equation 2.5, the received signal consists of the desired user's signal along with (K-1) undesired user's signals. For a single user channel the optimal receiver turns out to be a matched filter receiver. In case of a multi-user channel, it has been shown that the use of a matched filter results in MAI thus limiting the channel capacity of the system [14]. Verdu [14] has shown that a maximum likelihood multiuser receiver which uses a matched filter followed by a Viterbi algorithm is optimal in the sense that the channel capacity is not limited either by MAI or near-far effect. The performance of a matched filter for a multi-user system is analyzed in the next section. This section also discusses the limitations of a conventional receiver. The analysis of a conventional receiver is to provide a motivation for multi-user demodulation.

2.2 Performance of a conventional receiver for a DS-CDMA multi-user system

The performance of a conventional receiver is outlined in this section. This analysis is based on work by Pursley [19]. As in Equation 2.5, the received signal is given as,

$$r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t) \tag{2.6}$$

For a matched filter system, the received signal is mixed down to baseband and multiplied by the desired PN sequence. Demodulation of a PN sequence results in the wide-band signal collapsing into a narrow-band signal. This signal is then integrated over one bit-period. Assuming that the delay and phase information of all the users is known to the receiver, the decision statistic for user 1 is given by,

$$Z_1 = \int_{jT_b}^{(j+1)T_b} r(t)c_1(t)\cos(w_c t)dt \tag{2.7}$$

This receiver is a single-shot detector; the bit decisions estimates are based on an observation interval of one bit. For the rest of the analysis, we will assume that bit 0 of user 1 is under consideration. Substituting Equation 2.1 and Equation 2.5 in Equation 2.7, the decision statistic of the receiver is ,

$$Z_1 = \int_{t=0}^{T_b} \left[\left(\sum_{k=1}^{k=K} \sqrt{2P_k} c_k(t-\tau_k) b_k(t-\tau_k) \cos(w_c t + \phi_k) \right) + n(t) \right] c_1(t)\cos(w_c t)dt$$

which may be expressed as

$$Z_1 = D_1 + \eta + I_1 \tag{2.8}$$

The decision statistic consists of three terms: the desired signal, the multiple access interference and noise output. D_i is the desired signal of user 1, I_i is MAI due to the presence of other users in the cell and η is the thermal noise contribution.
The desired user contribution is,

$$\begin{aligned} D_1 &= \sqrt{2P_1} \int_{t=0}^{T_b} c_k^2(t) b_k(t) \cos^2(w_c t)dt \qquad (2.9)\\ & \sqrt{\frac{P_1}{2}} \int_{t=0}^{T_b} \left(\sum_{i=-\infty}^{i=\infty} b_{k,i}\pi\left(\frac{t-iT_b}{T_b}\right)\right)(1+\cos(2w_c t))\,dt \\ &\approx \sqrt{\frac{P_1}{2}} b_{1,0} T_b \end{aligned}$$

The noise term η, is given by,

$$\eta = \int_{t=0}^{t=T_b} n(t)c_1(t)\cos(w_c t)dt \tag{2.10}$$

It is assumed that n(t) is white Gaussian noise with two sided power spectral density $N_0/2$. The mean of η is,

$$\mu_\eta = E[\eta] = 0 \tag{2.11}$$

and the variance of η is,

$$\begin{aligned} var(\eta) &= E[(\eta - \mu_\eta)^2] = E[\eta^2] \\ &\approx \frac{N_0 T_b}{4} \end{aligned} \tag{2.12}$$

where var(x) means the variance of random variable x. The third statistic is MAI, and is,

$$I_i = \int_{t=0}^{T} \sum_{k=1,k\neq i}^{K} \sqrt{\frac{P_k}{2}} b_k c_k(t) \cos(\omega_c t + \phi_k) \cos(\omega_c t + \phi_i) dt \tag{2.13}$$

The MAI contribution to the decision statistic can be modeled as a zero-mean Gaussian random variable with variance σ^2. The variance of MAI is [19],

$$var(I_k) = \frac{N T_c^2}{6} \sum_{i=1,i\neq k}^{K} P_i \tag{2.14}$$

Since the noise process and MAI are independent, the total variance is the summation of the two variances and is,

$$\sigma_{tot}^2 = \frac{N T_c^2}{6} \sum_{k=1}^{K} P_k + \frac{N_0 T_b}{4} \tag{2.15}$$

Hence, the probability of bit error is given by,

$$P_e = Q\left(\frac{\sqrt{\frac{P_0}{2}} T_b}{\sqrt{\frac{N T_c^2}{6} \sum_{k=2}^{K} P_k + \frac{N_0 T_b}{4}}} \right) \tag{2.16}$$

For an interference limited system; the situation where $var(I_k) \gg var(\eta)$, the probability of error is given by,

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right) \tag{2.17}$$

Equation 2.17 suggests that the performance of a conventional receiver is limited by MAI irrespective of $\frac{E_b}{N_0}$. The above results suggest that one method of increasing the user capacity is to reduce MAI. This necessitates employing a multi-user detector at the central receiver. Thus the motivating factor for employing multi-user detection is to increase the number of users supported by a CDMA system.

2.3 Review of Earlier Work In Multi-User Detection

As outlined in 2.1, the performance of a conventional receiver is limited by MAI and near-far effect. A method to increase the capacity of the system is to employ multi-user detection. Multi-user receivers exploit the structure of MAI in CDMA systems achieving improved performance over the conventional receiver. The concept of multi-user detection for CDMA was proposed by Schneider [11]. Verdu [14] presented an optimal multiuser receiver for single-cell CDMA system in AWGN channel. This receiver was shown to be near-far resistant regardless of the relative transmitter power levels of the users and showed significant performance improvement over the conventional receiver. The optimal receiver has a bank of matched filters followed by a Viterbi decision algorithm. The Viterbi algorithm searches the trellis spanned by K users and hence the complexity of the algorithm is exponential in number of users. The complexity of the optimal receiver makes it impractical to implement such a receiver. Various sub-optimal algorithms have been presented in literature and the optimal receiver serves as a benchmark for performance comparison. The references [15, 16] delineate a number of ideas that are present in much of the ongoing research. The work by Verdu [14] triggered a new research effort on sub-optimal algorithms with a view to reduce the complexity of the receiver.

2.3.1 Concept and Techniques

The thrust of the ongoing research in the area of multi-user detection is to design low-complexity algorithms which cancel MAI and are also near-far resistant. In this section, the concept and general signal processing techniques that have been proposed to cancel MAI are presented. The multiuser detection techniques can be broadly classified as linear detection techniques and non-linear detection techniques.

Verdu presented an optimal multiuser receiver for a CDMA system. It was shown that an optimal receiver has a performance equal to a single user receiver operating in a single user channel[14]. The optimum receiver was shown to be near-far resistant regardless of the relative transmitter power levels of the users and showed significant performance improvement over the conventional receiver. Verdu also showed that the performance of a CDMA system is not limited by near-far resistance or MAI and that these are only the manifestations of using a correlation receiver. He also showed that the matched filter outputs for each user forms a set of sufficient statistics that, if processed properly, can lead to an optimal multiuser receiver. The optimal receiver consists of a bank of K matched filters followed by a maximum likelihood Viterbi decision algorithm. The optimal receiver proposed is shown in Fig. 2.2. The complexity of the optimal multiuser receiver precludes it from practical implementation, thus there is a need for design of sub-optimal algorithms. The sub-optimal algorithms designed should have a performance close to the optimal receiver with a complexity far less than the optimal receiver. A number of sub-optimal multiuser detectors have been proposed in the literature. The primary goal of the research is to reduce the complexity of the receivers while providing performance as close as possible to an optimal receiver. Most sub-optimal receivers have taken two basic forms, linear de-correlating receivers and non-linear receivers. A de-correlating detector for both the synchronous and asynchronous CDMA systems was developed by Lupas and Verdu [21], [22]. The linear de-correlating approach required the knowledge of the delays of the users and its performance was independent of the received powers of the users. The receiver is implemented by a set of linear transformations on the estimates at the output of the matched filter. This de-correlating detector is similar to the zero forcing equalizer in

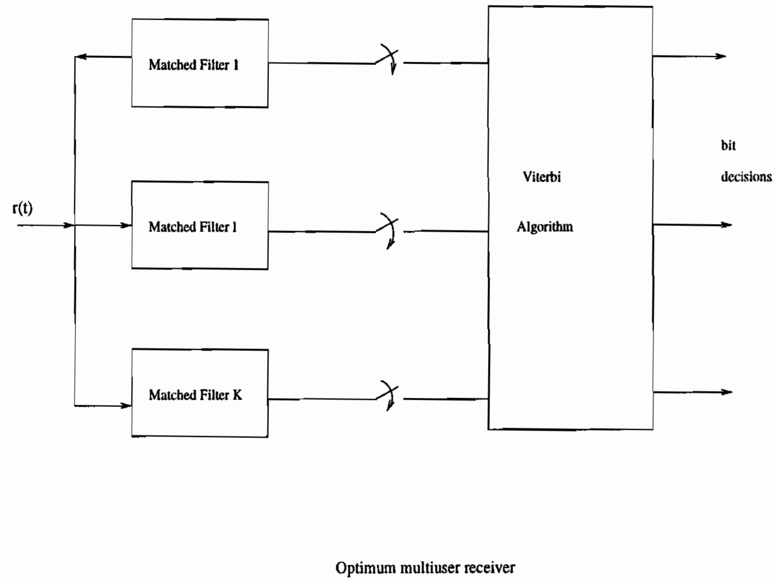


Figure 2.2: Optimal Multiuser Receiver

a single-user ISI channel. The performance of a de-correlating detector in high SNR environments is close to the optimal receiver but at low signal to noise ratios the enhancement in noise deteriorates its performance. From the implementation point, the linear transformation matrix requires the calculation of cross-correlations between the various signature sequences and to calculate them is fairly complex. Another approach to multi-user detection is to have a sequential decoder after the matched filters bank, this is similar to the sequential algorithm which is used for convolutional codes with a large constraint length. A sub-optimum sequential decoder for CDMA systems was presented by Xie *et al.* [23], with a complexity which is not exponential in the number of users. The performance of such a receiver was much worse than the optimal receiver when large number of users are present in the system. A family of sub-optimum detectors with minimum mean squared error (MMSE), weighted least squares (WLS) detection, linear feedback equalizers and decision feedback equalizers were developed and evaluated in [24]. A minimum mean square error (MMSE) detection scheme was discussed in [25], this scheme requires the transmission of training symbols. A blind multiuser MMSE detector was proposed by Honig *et al.* [26], which does not require training symbols. A tutorial on blind multiuser detection has been written by Madhow

[27]. All the schemes are linear detection techniques and the linear structure of these receivers limits the performance of these receivers.

A number of sub-optimal receivers have been proposed which are non-linear in nature. The multiuser receivers not only provide near-far resistance but also have been used with interference cancellation schemes. The interference cancellation receivers can be broadly categorized into,

- Parallel Interference cancellation (PIC) receivers
- Successive Interference cancellation (SIC) receivers

The concept of multi-stage parallel interference cancellation scheme was proposed by Varanasi and Aazhang in [28]. At each stage of the receiver, the receiver comprises either a bank of matched filters, a de-correlating type detector or a linear detector, is followed by an interference subtraction operation. The conventional detector in each stage provides estimates that is used to make a hard decision for the user, and assuming that these decisions are correct, the MAI is reconstructed and then cancelled from the desired signal. Figure 2.3 presents a block diagram of a multi-stage interference cancellation receiver for BPSK systems as proposed in [28]. Analysis and simulations were presented for two stages in these publications. As in the case of most multiuser receivers, these receivers also require knowledge of channel parameters like delays, phases and channel coefficients for perfect interference cancellation.

The successive interference cancellation scheme was first introduced by Cover [9] to achieve the rate pairs at the corner points of the capacity region. Viterbi [29] introduced the idea of successive cancellation for a CDMA system with power control, and was able to show that the capacity of such a system is close to the Shannon capacity. In successive cancellation scheme the interference is cancelled successively at each stage. The strongest user is estimated first and these estimates are used to subtract the strongest signal from the received signal and the residual signal is passed to the subsequent stage for processing. The process is repeated at every stage. For a K user system, this requires K-1 cancellations. The disadvantage of this scheme is the delay which is incurred due to the number of cancellations. Patel and Holtzman [30] showed

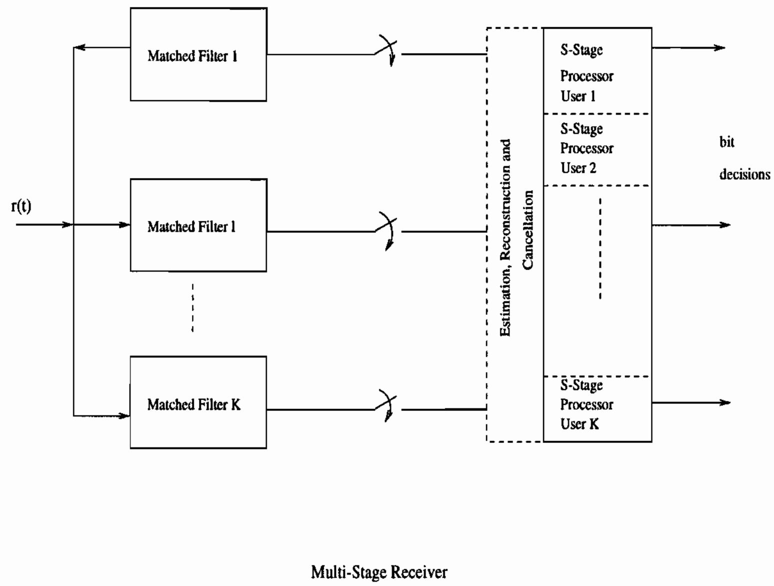


Figure 2.3: Multistage Receiver

the capacity improvement for a CDMA cellular system when a SIC is used. They used the correlator outputs to estimate the powers of the users in order to rank them and cancel accordingly, and a practical implementation of the SIC and its performance was discussed by [31].

In recent years, the trend has been to develop sub-optimal receivers which adapt to the varying channel parameters. Many sub-optimal detectors have been proposed which are adaptive in nature. Xie et al. [23] have extended their previous work and proposed a multiuser receiver that jointly detects the signal and the channel parameters. The adaptive receiver uses a binary-tree search algorithm for signal detection and a recursive least squares estimator to estimate the channel parameters. The complexity of the receiver was shown to be $O(K^2)$. Chen and Roy [32] have proposed an adaptive de-correlating multiuser receiver for synchronous CDMA over AWGN channels. An adaptive multiuser detector that uses a set of extended Kalman filters (EKF) to estimate the user delays and amplitudes is suggested in [33]. The authors use a set of cross correlation functions to update the estimator at the bit rate than the chip rate. A self organizing multiuser receiver is proposed in [34]. The receiver is capable of blind adaptation to the varying channel and uses neural networks to estimate the channel parameters. Holtzman *et al.*[17] and Moshavi [18] have written excellent tutorials on the various multiuser detection schemes.

Currently, a large part of the research in receiver design is concentrated on the use of multistage interference cancellation receivers. Although both successive and parallel interference cancellation receivers have been shown to outperform the conventional single user receiver, they differ from each other in performance and implementation. Parallel interference cancellation treats each user equally and simultaneously cancels interference from all interferers. The estimation and interference cancellation can be implemented in parallel and are suitable for real time implementation. Thus the delay involved in implementing the parallel interference cancellation is quite less compared to the successive interference cancellation receiver where the interferers have to be ranked before interference cancellation. Correal [35] discusses a DSP implementation of a parallel interference cancellation receiver, and a practical implementation of a successive interference cancellation receiver is discussed in [36].

Presently, multiuser receivers are proposed to be used only at the base station. It has been shown that for a CDMA cellular system, most of the time the reverse link limits the capacity of the system [37]. The base station has the advantage of the knowledge of all the user's PN codes which is inherently assumed in most of the multiuser detection schemes. In the forward link, each user's mobile has knowledge of its own PN code, hence implementation of a multiuser scheme is difficult. Some other issues which precludes the use of multiuser detection scheme is the complexity of the algorithm. In reality, the capacity gain that can be achieved by multiuser detection scheme are limited because of the other cell interference which is not cancelled by the base station.

2.4 Chapter Summary

The performance of a conventional receiver is severely limited by MAI and a method to overcome this problem is to employ a multiuser receiver at the base station. The complexity of the optimal receiver is exponential in the number of users and this fact has prompted researchers to develop sub-optimal receivers. The main objective of this technical report is to modify an existing receiver structure in order to develop a new receiver algorithm. The proposed new receiver structure should be practically implementable and should provide a significant capacity gain when compared to a conventional receiver. The two candidate receiver structures which are considered for modification are SIC and PIC schemes. The new receiver that is sought to be developed is to be based on a receiver structure that yields the maximum user capacity. Although, the performance of SIC and PIC receivers was presented in some of earlier works [30] [28]; no conclusions could be derived as to which receiver algorithm has better performance. An information theoretic approach will be developed in the next chapter for analyzing the user capacity of a CDMA system employing any receiver. Unfortunately, the performance of a CDMA system employing a PIC receiver could not be calculated by the information theoretic approach. In order to conclude as to which receiver algorithm supports a larger number of users, the SIC receiver's performance is compared to the performance of an optimal receiver. The information theoretic approach which is presented in the next chapter is used to address the question of bandwidth assignment for a CDMA system.

Chapter 3

Information Theoretic Approach To Evaluate Capacity

In general, the user capacity of a CDMA system employing an interference cancellation receiver is calculated using simulations. This approach has limitations if a generic comparison is to be performed between two interference cancellation receivers. In this chapter, an information theoretic approach is presented to evaluate the user capacity of a CDMA system irrespective of the receiver. The idea in developing such an approach is to evaluate the user capacities of both SIC and PIC receivers for a given QoS. The evaluation of the user capacities is performed in order to find out which IC scheme can support a larger number of users. A new receiver algorithm based on the IC scheme which supports a larger number of users is subsequently developed. The information theoretic approach couldn't be applied to a system with PIC receiver, hence the user capacity supported by a CDMA system employing a SIC receiver is compared with a CDMA system employing an optimal receiver. This comparison provides an insight into the performance of a SIC receiver vis-a-vis a PIC receiver.

This approach is also used to address the problem of bandwidth assignment for a CDMA system. As already stated, for a DS-CDMA system the narrow-band data signal is spread to a wider bandwidth. The wide-band signal requires a larger bandwidth than the narrow-band signal and the additional bandwidth required to transmit the wide-band signal is termed as bandwidth expansion. Modulating the data signal by a wide-band PN/chip sequence results in the signal spectrum expansion. Another method to expand the bandwidth is to apply error control coding to the data signal. Error control coding results in bandwidth expansion because of addition of redundancy to the signal. Thus, for a DS-CDMA system, bandwidth expansion could be due to modulation by PN codes or due to application of error control coding. Is there a bandwidth allocation/assignment which results in the system supporting maximum user capacity is the problem that is sought to be answered. A number of papers have addressed this issue for a spread-spectrum system [38, 39, 40, 41, 42]. The background and related work are presented in more detail in section 3.1. A generic cellular system model is presented in section 3.2. In sections 3.5 and 3.6, the performances of a conventional and a SIC receiver are analyzed. Section 3.7 presents the user capacity analysis for a multiple cell system.

3.1 Introduction

As stated earlier, for a spread spectrum system, bandwidth expansion could be due to modulation by PN codes or due to error control coding. The problem to be investigated is whether the user capacity of the system depends on the bandwidth allocation; if so is there any allocation which maximizes the user capacity. Two extreme cases of bandwidth assignment are considered in this chapter. If the bandwidth expansion is due to use of only error control codes it is defined as bandwidth expansion due to complete error control coding. Similarly if the bandwidth expansion is due to modulation by a PN code, it is defined as no error control coding case. The above issue was first addressed by Hui [38], who analyzed the capacity of a spread-spectrum system for a binary symmetric channel (BSC) with a cross-over probability p. The cross-over probability for a CDMA system depends on the number of users in the system. Hui showed that the the channel has maximum throughput when the bandwidth allocation is completely as a result of error control coding. Viterbi [29] showed that a spread-spectrum system with an interference cancellation receiver achieves capacity close to Shannon's channel capacity when the spreading is completely due to error control coding. Veeravalli and Aazhang [39] addressed this issue for a conventional receiver and also for an interference cancellation receiver. Shamai and Wyner [40, 41] addressed this issue comprehensively for cellular systems. The results presented in this chapter are similar to some results presented in [40, 41].

In the proposed approach, the user capacity issue is addressed from an information theoretic view. Channel capacity is used to evaluate the number of users a CDMA system can support for a given bandwidth. Channel capacity is the maximum rate at which information can be transmitted with an arbitrarily low probability of error and addressing the capacity of a CDMA system from Shannon's sense yields the theoretical maximum number of users a CDMA system can support for a given bandwidth. For the analysis of a conventional receiver we assume a simple model where all other users are assumed to be causing interference to the desired user. The same model is then extended to a SIC receiver. In a CDMA system the frequency re-use factor is one thus all out-of-cell users in addition to in-cell users cause interference to the in-cell user. For a multiple-cell system the other cell multi-user interference is defined as a ratio of the in-cell multi-user interference. This argument was used by Viterbi to bound the maximum achievable capacity gain of an interference cancellation receiver [44]. In case of a single-cell system, the capacity gain for an optimal interference cancellation receiver is unbounded. It is assumed that the interference cancellation receiver is not able to cancel out-of-cell interference. Under this assumption, for a multi-cell system, the capacity gain is bounded by the un-cancelled out of cell interference.

Let λ be the ratio of the total average other-cell user interference to average same-cell other user interference. The total interference that is seen by any user in a multiple cell system is increased by a factor of λ when compared to a single cell system. Consider the generic cellular system model shown in Fig. 3.1. Each cell is surrounded by six adjacent cells and assume equal loading of K users in each cell. The in-cell interference seen by any user is,

$$I_{in-cell} = \frac{(K-1)}{N} P_{in-cell} \tag{3.1}$$

where, N is the processing gain. For the hexagonal cell system shown in 3.1, λ is

$$\lambda = \frac{6KP_{out_of_cell}}{(K-1)P_{in-cell}} \tag{3.2}$$

For a multiple-cell system, the total interference that is seen by the desired user is,

$$I_{total} = (1+\lambda)\frac{(K-1)}{N}P_{in-cell} \tag{3.3}$$

The capacity of the system is inversely proportional to total interference

$$C \propto \frac{1}{Interference} \tag{3.4}$$

An optimal interference cancellation receiver cancels all in-cell interference. The residual interference is λ times the in-cell interference. Hence the capacity of system with an optimal IC receiver is,

$$C_{optimal\,ic} \propto \frac{1}{\lambda I_{in-cell}} \tag{3.5}$$

For a system with no interference cancellation receiver, the capacity is,

$$C_{no\,ic} \propto \frac{1}{(1+\lambda)\,I_{in-cell}} \tag{3.6}$$

The capacity gain that is obtained when an optimal IC is employed in the system is,

$$Gain = \frac{(1+\lambda)}{\lambda} \tag{3.7}$$

This is the theoretical maximum gain that can be obtained by an IC receiver. This serves as the benchmark to compare the performance of any IC receiver.

3.2 Multi-Cell System Model

A multi-cell CDMA system and the terms used in this chapter are presented in this section.

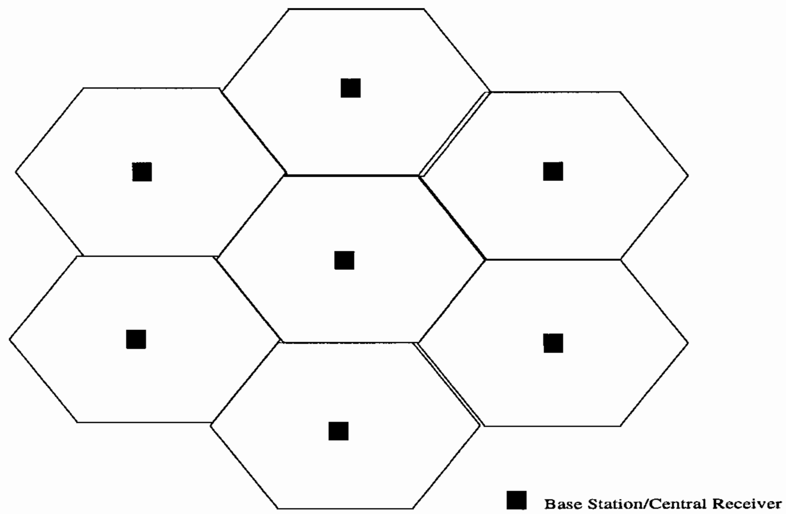
A cellular system consists of a base station which is connected to the switching center. A general cellular system model is shown in Fig. 3.1. All the users in a given cell transmit information to the base station over a wireless link. As the spectrum is limited it is shared amongst a cluster of cells. The cluster size defines a term called frequency re-use factor. If the cluster size is N, the whole spectrum is re-used after N cells and hence the frequency re-use factor is $\frac{1}{N}$. For a DS-CDMA multiple cell system, a frequency re-use factor of one is assumed. A frequency re-use factor of one suggests that the whole spectrum is used in all the adjacent cells, thus out-of-cell users cause additional interference to the in-cell desired user. Experimental results have shown that the value of λ is around 0.5 [44].

Let there be K active users in each cell operating at the same data rate R_d. Assume that the system has a total bandwidth W, then the total bandwidth expansion is $G = \frac{W}{R_d}$ where G denotes the bandwidth expansion. The value of G depends on the error control code-rate and chip rate. The dependence of G on error control code-rate and chip rate can be understood by describing the encoding process. The encoding process for each user is shown in Fig. 3.2. Denoting the error control code rate as R_c results in a symbol-rate $R_s = \frac{R_d}{R_c}$. These symbols are then modulated by a PN code at the chip rate. Defining the processing gain G_p as ratio of the PN code rate to the symbol rate, results in $G_p = \frac{c}{R_s}$, where c is the chip-rate. Assuming a spectrum efficiency of 1 chip/sec/Hz results in a total bandwidth expansion $G = \frac{G_p}{R_c}$. As $R_c \in [1, \frac{1}{G}]$; $G_p \in [G, 1]$. Before, presenting the information theoretic approach, the concept of channel capacity is discussed.

3.3 Concept of Channel Capacity

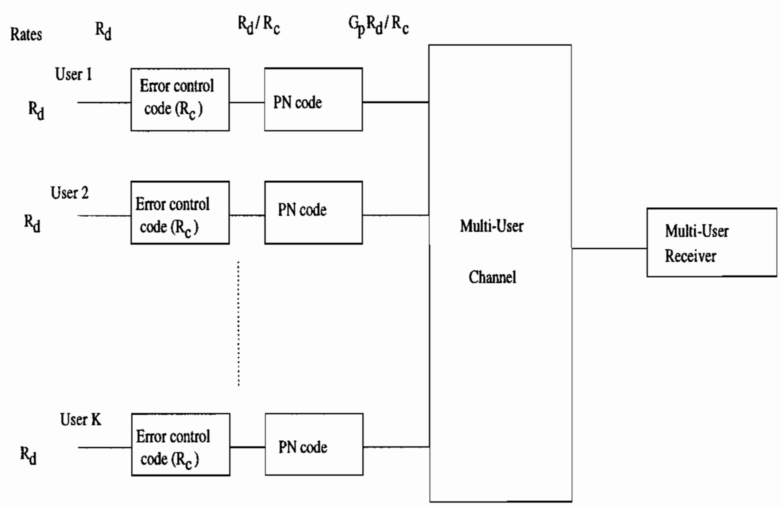
Channel capacity for a multi-user network is presented in this section. Shannon in 1948 showed that information could be transmitted at any rate below the capacity of the channel with an arbitrarily low probability of error. Channel capacity for a band-limited AWGN channel is ,

$$C = W\log_2\left(1 + \frac{P}{N_0 W}\right) \text{ bits/sec} \tag{3.8}$$



Cellular System Model

Figure 3.1: Cellular System Model



System Model

Figure 3.2: Encoding Process

where W is the channel bandwidth, P is the signal power and $\frac{N_0}{2}$ is the two-sided noise power spectral density. For a multi-user network, the channel capacity is described by a K-tuple, where each point denotes the rate achieved by that user. In a multi-user network, the achievable rate-region is determined by the following inequalities,

$$\begin{aligned} R_1 &\leq W\log_2\left(1+\frac{P_1}{N_0W}\right) \\ R_2+R_1 &\leq W\log_2\left(1+\frac{P_2+P_1}{N_0W}\right) \\ \sum_{i=1}^{K} R_i &\leq W\log_2\left(1+\frac{\sum_{i=1}^{K}P_i}{N_0W}\right) \end{aligned} \tag{3.9}$$

where $P_1, P_2, \ldots P_K$ are the signal powers of the users. In an equal rate case, the last inequality dominates the others [10]. Only the equal data rate is considered in this chapter.

3.4 Successive Interference Cancellation Scheme

Cover first introduced the concept of successive interference cancellation in [9]. Cover used the idea of successive cancellation to achieve the rate pairs at the corner points of the capacity region. The successive cancellation idea along with power control was used by Viterbi in [29] to achieve equal rates for all users.

In a multiple access system, if different users are received at varying signal power levels, the difference in received signal power levels can be utilized to achieve equal rates for all the users. Under the assumption that the received signal statistics are Gaussian, the user with the highest received signal power is decoded first by treating all the other signals as noise. After decoding the strongest received signal, it is re-encoded and subtracted from the received signal. The signal with the highest power in the residual signal is then decoded, re-encoded and subtracted from the received signal. This process is repeated at every stage.

Consider the example of two users with different power levels P_1 and P_2 where $P_2 > P_1$. The user with the highest received power can now be decoded with a low probability of error if $R_2 < C\left(\frac{P_2}{P_1+N_0}\right)$, where $C(x) = \log_2(1+x)$. As previously stated, this is simply the Shannon channel coding theorem for the AWGN channel. A new signal is formed by subtracting the decoded user 2's signal from the received signal, and the rate achieved by user 1 is $R_1 < C\left(\frac{P_1}{N_0}\right)$. For a system which supports equal rates for all users, [9]

$$\frac{P_2}{N_0} = \frac{P_1}{N_0}\left(1 + \frac{P_1}{N_0}\right) \tag{3.10}$$

The above equations suggest that the decoding process works perfectly if the received powers are different. The above idea can be easily extended to a K-user system. A SIC receiver is shown in Fig. 3.3.

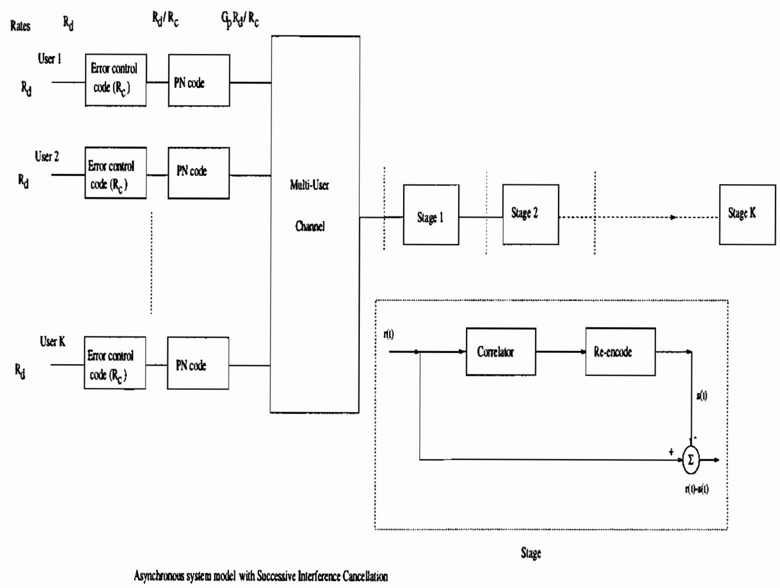


Figure 3.3: Successive Interference Cancellation Receiver

3.5 Capacity of a Single Cell DS-CDMA with a Conventional Receiver

The user capacity of a DS-CDMA system with a conventional receiver is evaluated in this section. A synchronous system with orthogonal codes is the first system that is considered for capacity evaluation. The synchronous system is considered to reiterate the fact that an orthogonal CDMA system achieves the same capacity as any other orthogonal system, for example a FDMA or a TDMA system.

3.5.1 Synchronous System with Orthogonal Codes

For a synchronous DS-CDMA system to be orthogonal the cross-correlations between the various codes should be zero. This implies that the PN codes that are chosen should be mutually orthogonal. As the cross-correlations are zero there is no MAI. For a processing gain G_p it turns out that the number of mutually orthogonal sequences that can be generated is equal to G_p. This is under the assumption that the alphabet of the sequences is binary. Thus the number of users the system can support $K = G_p$ users. The rate achieved by any user from Equation 3.9 is,

$$\begin{aligned} R_d &\leq \frac{W}{G_p}\log_2\left[1 + \frac{P}{N_0\frac{W}{G_p}}\right] \\ &\leq \frac{W}{G_p}\log_2\left[1 + \frac{PG_p}{N_0 W}\right] \end{aligned} \tag{3.11}$$

The received signal is passed through a correlator. Viewing the operation in frequency domain, the correlator collapses the spectrum of the user from wide-band to a narrow-band signal. For a channel bandwidth W and processing gain G_p, the spectrum is collapsed from from W to $\frac{W}{G_p}$, which is defined as the data bandwidth or information bandwidth. P refers to the received signal power where, $P = E_b R_d$. E_b is the bit-energy density. Substituting for G_p in Equation 3.11,

$$\begin{aligned} R_d &\leq \frac{W}{K}\log_2\left[1 + \frac{KP}{N_0 W}\right] \\ &= \frac{W}{K}\log_2\left[1 + \frac{KE_b R_d}{N_0 W}\right] \end{aligned} \tag{3.12}$$

re-writing Equation 3.12, in terms of the bit-energy to noise density ratio,

$$\frac{E_b}{N_0} = \left(\frac{2^{\left(\frac{KR_d}{W}\right)} - 1}{\frac{KR_d}{W}}\right) \tag{3.13}$$

For an FDMA system with a total bandwidth W and a channel bandwidth $\frac{W}{K}$ the bit-energy to noise density ratio is,

$$\frac{E_b}{N_0} = \left(\frac{2^{\left(\frac{KR_d}{W}\right)}-1}{\frac{KR_d}{W}}\right) \tag{3.14}$$

The result confirms the fact that theoretically all the three orthogonal systems achieve the same capacity.

3.5.2 Capacity of Conventional Receiver for an Asynchronous System (AWGN Channel)

For an asynchronous system, a set of mutually orthogonal sequences is difficult to be designed. Due to the asynchronous nature, a mutually orthogonal system designed for a particular known set of delays, ceases to be mutually orthogonal for a different set of delays. Hence an asynchronous CDMA system results in a non-orthogonal system. For a non-orthogonal system the cross-correlations between the code-sequences is non-zero thus resulting in MAI.

The total noise at the output of the conventional receiver consists of MAI plus thermal noise. The spectrum at the output of the correlator is shown in Fig. 3.4. The equivalent noise power $\tilde{N}_0$ at the correlator output is,

$$\tilde{N}_0 = N_0 b + (K-1)P\frac{b}{W}$$

where b is the information/data bandwidth and P is the signal power for any user. The rate achieved by any user in the system is,

$$R_d \le b\log_2\left(1+\frac{P}{\tilde{N}_0}\right) \tag{3.15}$$

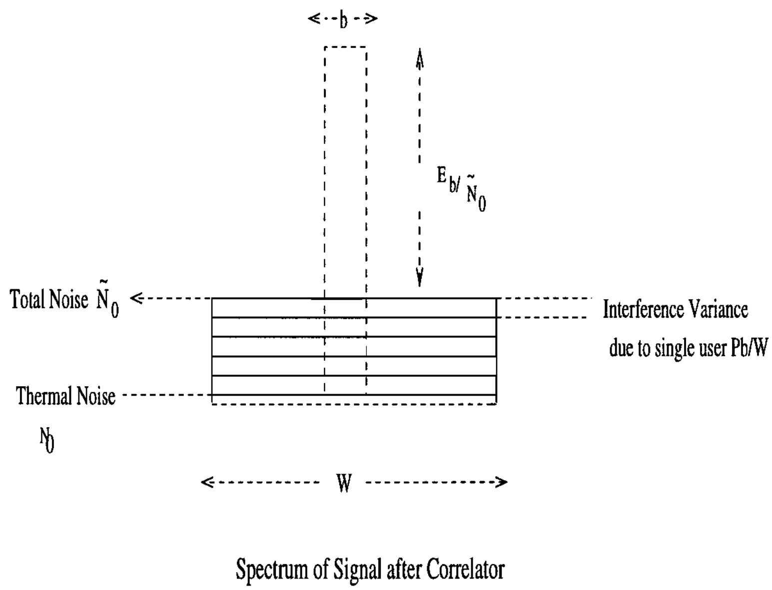


Figure 3.4: Spectrum of signal

substituting for $b = \frac{W}{G_p}$, $P = E_b R_d$, and for $\tilde{N}_0$, in Equation 3.15,

$$\begin{aligned} R_d &\leq \frac{W}{G_p}\log_2\left[1+\frac{E_b R_d}{\frac{N_0 W}{G_p}+(K-1)\frac{E_b R_d}{G_p}}\right] \\ &\leq \frac{W}{G_p}\log_2\left[1+\frac{\frac{E_b R_d G_p}{N_0 W}}{1+(K-1)\frac{E_b R_d}{N_0 W}}\right] \end{aligned} \tag{3.16}$$

The rate achieved by any user in the system is given by Equation 3.16. The number of users in the system are,

$$K \leq 1 + G\left[\left(\frac{2^{\frac{R_d G_p}{W}}-1}{\frac{R_d G_p}{W}}\right)^{-1} - \left(\frac{E_b}{N_0}\right)^{-1}\right]$$

Let

$$\gamma_{req} = \left(\frac{2^{\frac{R_d G_p}{W}}-1}{\frac{R_d G_p}{W}}\right) \tag{3.17}$$

substituting for γ_{req} in Equation 3.17

$$K \leq 1 + G\left[\frac{1}{\gamma_{req}} - \frac{1}{\frac{E_b}{N_0}}\right] \tag{3.18}$$

Equation 3.18 is similar to the capacity equation in Gilhousen *et al* [37]. Equation 3.18, evaluates the capacity of the system as a function of the bandwidth expansion G which depends on G_p and $\frac{E_b}{N_0}$. The limits on G_p correspond to the extreme cases of bandwidth expansion. $G_p = 1$, refers to the case when the bandwidth expansion is only by error-control coding. $G_p = G$, represents the case when the bandwidth expansion is completely due to PN codes. The value of γ_{req} for both the cases is,

$$\gamma_{req} = \begin{cases} 1 & \text{when } G_p = G \\ \ln 2 & \text{when } G_p = 1 \end{cases}$$

From Equation 3.18,

$$\begin{aligned} K &\leq 1 + G\left(1 - \frac{1}{\frac{E_b}{N_0}}\right) \quad G_p = G \\ &\leq 1 + G\left(\frac{1}{\ln 2} - \frac{1}{\frac{E_b}{N_0}}\right) \quad G_p = 1 \end{aligned} \tag{3.19}$$

Comparing the capacities for $G_p = 1$ and $G_p = G$ results in,

$$C_{ba} = \frac{\left(\frac{\frac{1}{G}}{2^{1/G} - 1} - 1\right)}{1 + \frac{1}{G} - \frac{1}{\frac{E_b}{N_0}}} \tag{3.20}$$

where, C_{ba} is the capacity gain due to bandwidth allocation. For large G and large $\frac{E_b}{N_0}$ a capacity gain of 44 percent is obtained compared to the no error control coding case. Fig. 3.5 shows the capacity gain for various values of G and $\frac{E_b}{N_0}$. For large $\frac{E_b}{N_0}$ and large G, the capacity increase for $G_p = 1$ relative to $G_p = G$ is around 44 %. This increase is obtained at the cost of complexity of the receiver, since a receiver with an error control decoder is complex than a simple correlator.

3.5.3 Asynchronous System for Rayleigh Fading Channel

In this section, capacity of a DS-CDMA system in a Rayleigh fading environment is evaluated. This analysis considers a conventional receiver. For a Rayleigh fading channel, the achievable rate in an average sense can be obtained [45]. Assume that each user

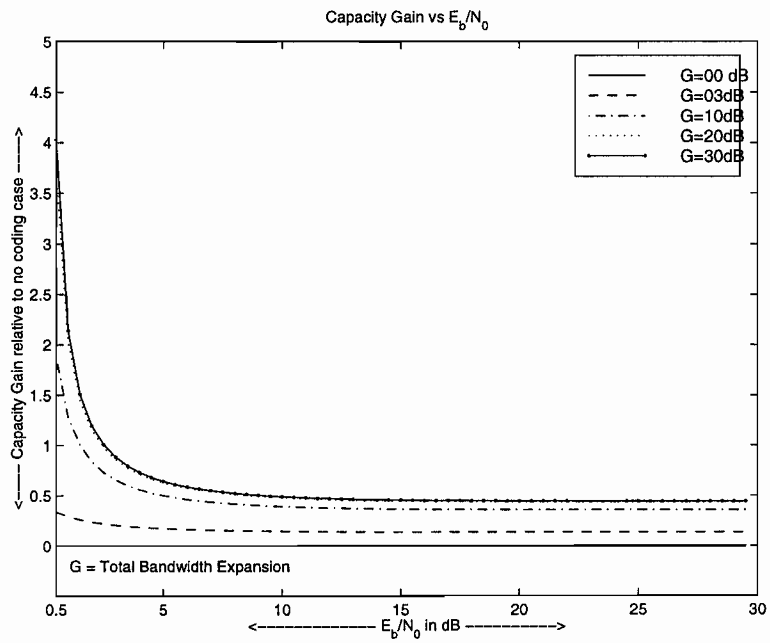


Figure 3.5: Capacity Gain due to bandwidth allocation

on an average achieves equal data rate. For the reverse link, since each user transmits asynchronously, independent fading of all the received signals can be assumed. Let the average power be the same for all signals. The average rate achieved by any user is,

$$\widehat{R}_d = W\int_0^\infty \log_2(1+\xi)\, P_\xi(\xi) d\xi \tag{3.21}$$

where, $P_\xi(\xi)$, is the probability density function (pdf) of ξ. The random variable (r.v) ξ, can be written from Equation 3.16, as follows,

$$\xi = G_p \frac{\gamma}{1+\sum_{i=1}^{K-1}\gamma_i} \tag{3.22}$$

where, γ is the instantaneous signal power of each user and is a r.v. For a Rayleigh fading channel, γ has a pdf [46],

$$P_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{\frac{-\gamma}{\bar{\gamma}}} \tag{3.23}$$

where, $\bar{\gamma}$ is the average signal power. The pdf of γ is exponential with mean $\bar{\gamma}$. To obtain the average rate achieved by any user in the system, Equation 3.21 has to be evaluated; for which the pdf of ξ is to be evaluated.

The distribution of the numerator in Equation 3.22 is known and is an exponential pdf. The pdf of sum of (K-1) exponentially distributed random variables is to be evaluated. The pdf of an exponential distribution can be written in terms of a Gamma distribution. The Gamma distribution is [47]

$$\begin{aligned} P_X(x) &= \frac{x^{\lambda-1}e^{-\frac{x}{\alpha}}}{\Gamma(\lambda)\alpha^{\lambda}} \quad 0 \leq x \leq \infty \\ &= 0 \qquad\qquad\quad x < 0 \end{aligned}$$

The Gamma distribution has two free parameters (λ, α). The pdf of exponentially distributed r.v. γ when written in terms of Gamma distribution has the following free parameters $(1, \bar{\gamma})$.

If $Z_1, Z_2, \ldots, Z_K$, are K independent r.v's with Gamma distribution having free parameters (λ_1, α), (λ_2, α), ... (λ_K, α), then $\sum_{i=1}^{K} Z_i$, is also Gamma distributed with parameters, $\lambda = \sum_{i=1}^{K} \lambda_i$ and $\alpha = \alpha$ [47]. The sum of (K-1) independent exponentially distributed r.v's also has a Gamma distribution with free parameters (K-1, $\bar{\gamma}$). Let Y be another random variable, where

$$Y = 1 + \sum_{i=1}^{K-1} \gamma_i$$

Since Y is a sum of exponential r.v's Y has a Gamma distribution with parameters (K-1, $\bar{\gamma}$). The pdf of Y is,

$$\begin{aligned} P_Y(y) &= \frac{y^{K-2}e^{-\frac{y}{\bar{\gamma}}}}{\Gamma(K-1)\bar{\gamma}^{K-1}} \quad 0 \leq y \leq \infty \\ &= \qquad 0 \qquad\qquad y < 0 \end{aligned}$$

Let $M = \frac{\xi}{G_p}$ be another r.v. M can be written as $M = \frac{\gamma}{Y}$, where γ and Y have Gamma distribution with parameters $(1, \bar{\gamma})$ and (K-1, $\bar{\gamma}$) respectively. For two Gamma distributed r.v's V and W with parameters (λ_1, α) and (λ_2, α), then V/W is a r.v with a Beta distribution [47], with parameters λ_1 and λ_2. The Beta distribution is as follows

[48],

$$P_U(u) = \frac{u^{\lambda_1-1}(1+u)^{-\lambda_1-\lambda_2}}{B(\lambda_1,\lambda_2)} \tag{3.24}$$

where, B(x,y) is the beta function (Euler's integral of the first kind) [48], and is

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$$

The beta function can also be written as,

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Hence M is Beta distributed with parameters (1,K-1), and the probability density function of M is,

$$P_M(m) = \frac{(1+m)^{-K}}{B(1,K-1)} \tag{3.25}$$

since, $M = \frac{\xi}{G_p}$, the probability density function of ξ is,

$$\begin{aligned} P_\xi(\xi) &= \left[\frac{P_M(m)}{G_p}\right]_{m=\frac{\xi}{G_p}} \\ &= \frac{\left(1+\frac{\xi}{G_p}\right)^{-K}}{G_pB(1,K-1)} \end{aligned} \tag{3.26}$$

It can be noticed that the pdf of ξ is independent of $\bar{\gamma}$, the average signal power and is only a function of processing gain G_p and number of users K. Thus, for a Rayleigh fading channel the average rate achieved by any user in a DS-CDMA system with a conventional receiver is independent of signal power.

The average rate achieved by any user can be evaluated from Equation 3.21, and is

$$\begin{aligned} \widehat{R_d} &= W\int_0^\infty \log_2(1+\xi)\,P_\xi(\xi)d\xi \\ &= \frac{W}{G_p}\int_0^\infty \log_2(1+\xi)\,\frac{\left(1+\frac{\xi}{G_p}\right)^{-K}}{B(1,K-1)}d\xi \end{aligned} \tag{3.27}$$

The integral can be written in the form of a series [49]

$$\widehat{R_d} = \frac{W}{\log 2}(G_p^{2K-2})\left(\frac{1}{K-1} - \frac{(1-G_p)}{K}{}_3F_2(2,1,1;2,K+1;1-G_p)\right) \tag{3.28}$$

where, ${}_pF_q(\alpha_1, \alpha_2, \ldots \alpha_p, \beta_1, \beta_2 \ldots \beta_q; z)$ is a generalized hyper-geometric series [48]. Expanding Equation 3.28 results in,

$$\widehat{R}_d = \frac{W}{\log_2}(G_p^{2K-2})\left[\frac{1}{K-1} - (1-G_p)\sum_{l=0}^{\infty}(-1)^l B(l+1,K)(G_p-1)^l\right] \tag{3.29}$$

For the case of $G_p = 1$, substituting in Equation 3.29 results in,

$$\widehat{R_d} = \frac{W}{\ln(2)(K-1)} \tag{3.30}$$

and re-writing the above equation for number of users K;

$$K = 1 + \frac{W}{\ln 2\widehat{R_d}} \tag{3.31}$$

assuming that $\widehat{R_d} = R_d$ results in $\frac{W}{R_d} = G$, and substituting this in Equation 3.31, results in,

$$K = 1 + \frac{G}{\ln(2)} \tag{3.32}$$

The average number of users that a CDMA system can support in a Rayleigh fading channel is equal to the number of users the system can support in an AWGN channel when the bandwidth expansion is completely due to error-control coding. Fig. 3.6., shows the capacity of an AWGN system and a Rayleigh fading system as a function of the processing gain. It can be noticed that as the processing gain increases, the difference in the capacities of AWGN system and a Rayleigh fading system increases. For both the channels system capacities decrease as a function of the processing gain.

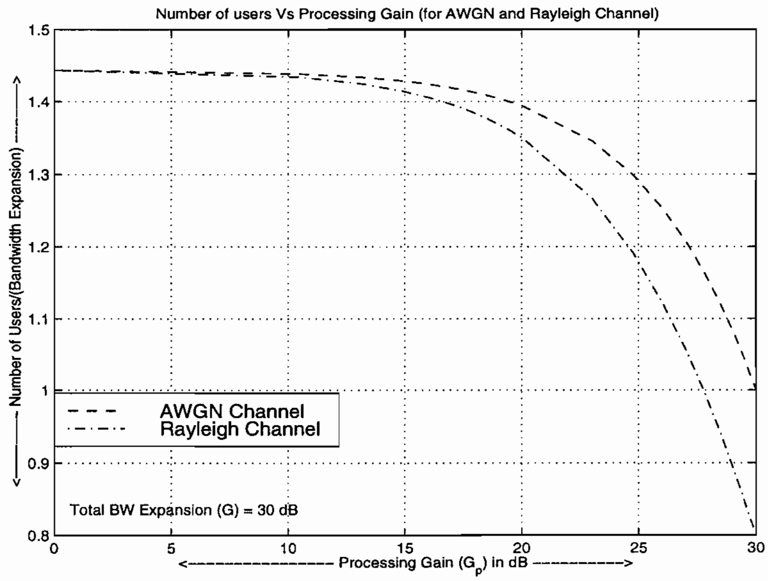


Figure 3.6: Capacity of DS-CDMA system for AWGN and Rayleigh Channel

3.6 Capacity of an SIC Receiver for a single cell CDMA system (AWGN Channel)

In this section, capacity of an asynchronous DS-CDMA system which employs a SIC receiver is evaluated. For perfect interference cancellation with a SIC receiver it is necessary that the signal powers be different. For the equal-rate case, the received signal powers should be in geometric progression [29]. Let $P_1 < P_2 < P_3 < \ldots < P_K$, where P_i denotes the signal power of user i. Let $\gamma_i = \frac{P_i}{N_0 W}$, be the signal to noise ratio of user i. The rate achieved by each user is,

$$\begin{aligned} R_K &\leq \frac{W}{G_p}\log_2\left[1 + \frac{\gamma_K G_p}{1 + \sum_{i=1}^{K-1}\gamma_i}\right] \\ &\vdots \\ R_2 &\leq \frac{W}{G_p}\log_2\left[1 + \frac{\gamma_2 G_p}{1 + \gamma_1}\right] \\ R_1 &\leq \frac{W}{G_p}\log_2\left[1 + \gamma_1 G_p\right] \end{aligned} \tag{3.33}$$

For the equal rate situation, $R_1 = R_2 = \ldots = R_K$, we obtain,

$$\begin{aligned}\gamma_2 &= \gamma_1 (1+\gamma_1) \\ &\vdots \\ \gamma_K &= \gamma_1 (1+\gamma_1)^{(K-1)}\end{aligned}$$

Equation 3.34 suggests that for perfect interference cancellation, the signal powers need to be in geometric progression. From Equation 3.34,

$$\gamma_1 = \frac{\left[2^{\left(\frac{G_p}{G}\right)} - 1\right]}{G_p} \tag{3.34}$$

$$\left(\frac{E_b}{N_0}\right)_1 = \frac{\left[2^{\left(\frac{G_p}{G}\right)} - 1\right]}{\frac{G_p}{G}} \tag{3.35}$$

The above equations suggest that the capacity of the system is not bounded. The maximum allowable signal power limits the maximum user capacity that can be supported by the system. For a fixed maximum signal power, Equation 3.35 suggests that the capacity is maximized when $G_p = 1$.

The capacities of the systems with SIC and without IC can be compared for a given allowable $\frac{E_b}{N_0}$. Since, for a SIC system the user's powers are in geometric progression, a criteria for comparing both the systems needs to be specified. Two contending but equally valid criteria for comparing both the systems are considered. The first criteria compares the mean $\left(\frac{E_b}{N_0}\right)$ of both the systems. This comparison equates the aggregate signal power in both the systems. The second criteria is to compare the $\left(\frac{E_b}{N_0}\right)$ of the user with the highest power in SIC system with $\frac{E_b}{N_0}$ of any user in system with conventional receiver. For this comparison case, the users in SIC system on an average expend less battery power than a user in a system without IC.

The mean $\left(\frac{E_b}{N_0}\right)$ of an SIC system is,

$$\left(\frac{E_b}{N_0}\right)_{mean} = \frac{\left[1 + \left(\frac{E_b}{N_0}\right)_1 \left(\frac{1}{G}\right)\right]^K - 1}{K\left[\frac{1}{G}\right]} \tag{3.36}$$

Figs. 3.7 and 3.8, compare the capacities of both the systems as a function of $\frac{E_b}{N_0}$, for $G_p = 1$ and $G_p = G$. In the figures, IC - Mean refers to the number of users supported when the means of both the systems are compared. IC-Highest user refers to the comparison of highest user's signal power for SIC system with any user's power for a conventional receiver. From Figs. 3.7 and 3.8, for SIC and a conventional receiver, the capacity is maximized when spreading is completely due to error control coding.

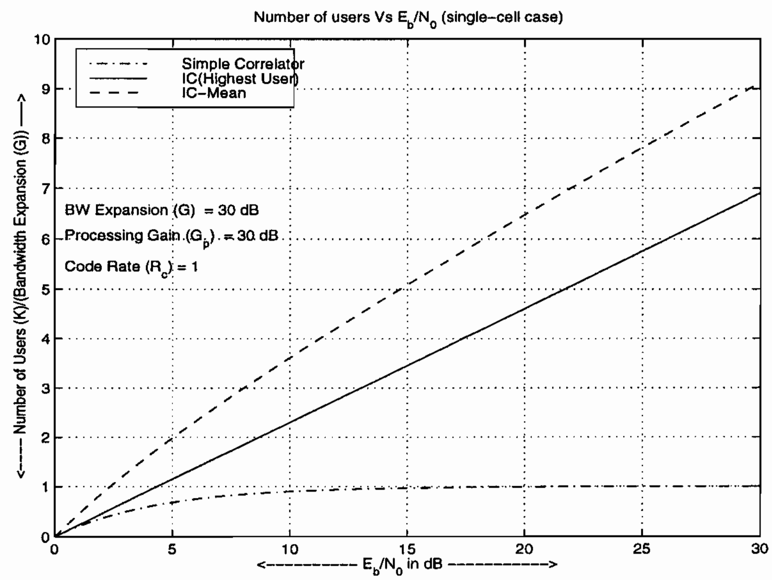


Figure 3.7: Number of users vs $\frac{E_b}{N_0}$, for $G_p = G$

The capacity gain obtained by the use of an interference canceler is shown in Figs.3.9 and 3.10, for the two extreme cases of bandwidth expansion.

The variation of the capacity as a function of the processing gain G_p for a fixed $\frac{E_b}{N_0}$ is shown in Figs.3.11 and 3.12. The results show that for a single-cell system, the capacity gain of an SIC system over a conventional system, increases with $\frac{E_b}{N_0}$.

3.7 Capacity Analysis For A Multiple-Cell System

In this section the performance of a conventional and a SIC receiver for an asynchronous DS-CDMA system in a multiple-cell system is analyzed. As stated earlier, the out-of-cell interference is modeled as a fraction of in-cell interference. Let the fraction of

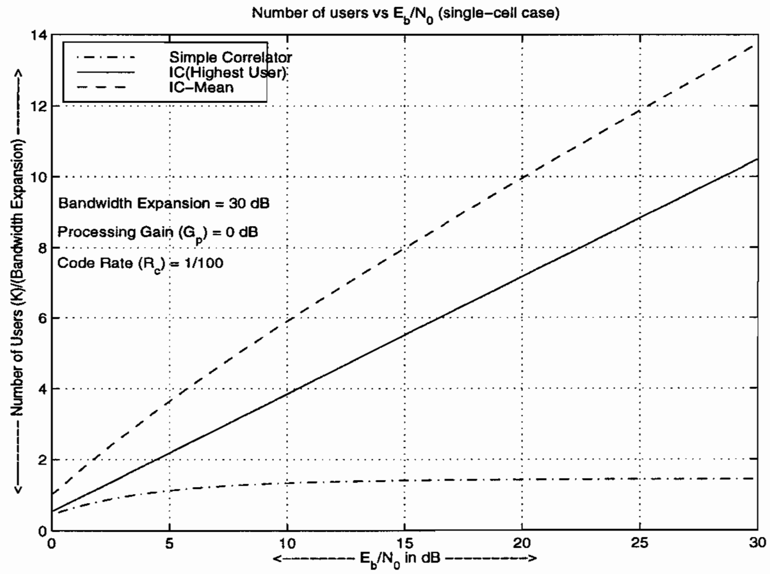


Figure 3.8: Number of users vs $\frac{E_b}{N_0}$, for $G_p = 1$

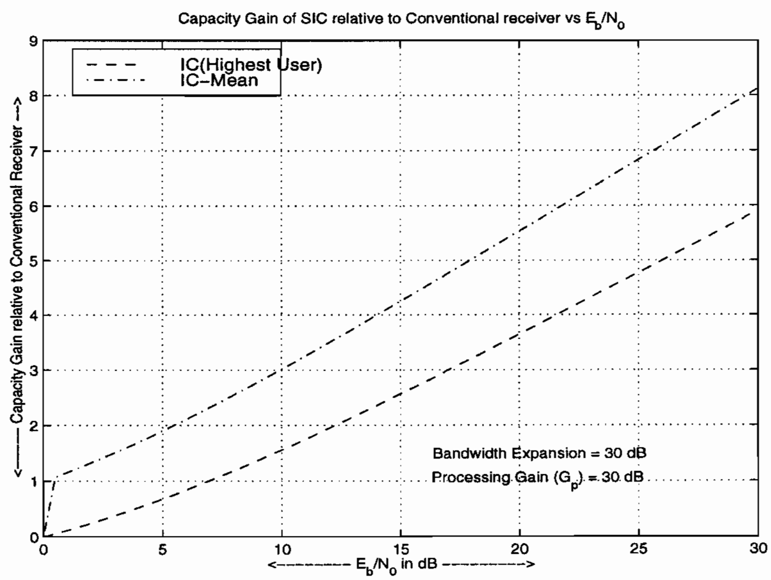


Figure 3.9: Capacity Gain vs $\frac{E_b}{N_0}$ for $G_p = G$

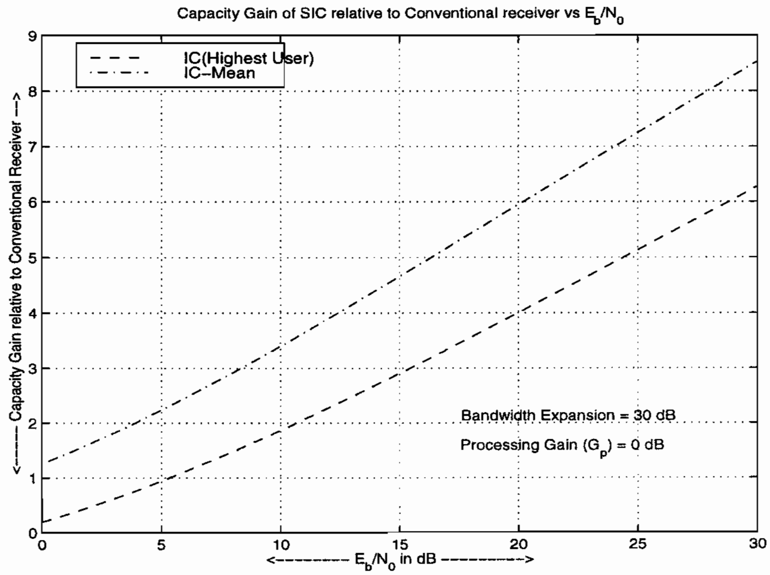


Figure 3.10: Capacity Gain vs $\frac{E_b}{N_0}$, for $G_p = 1$

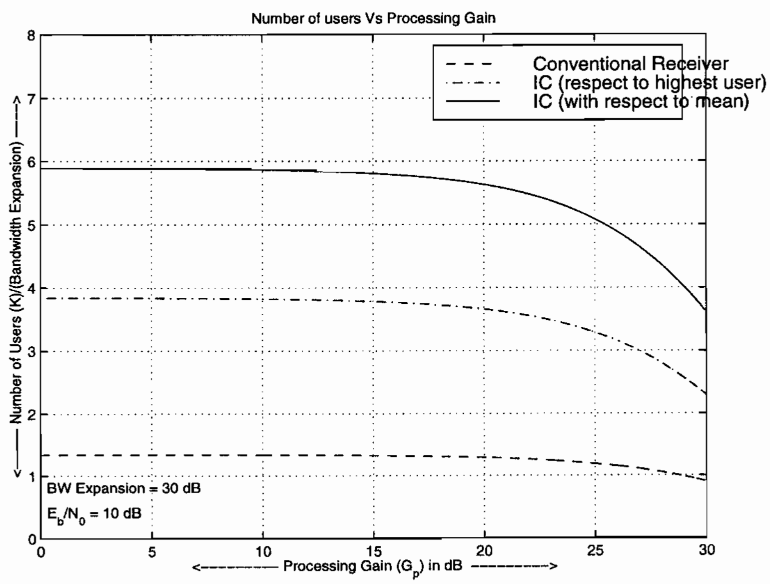


Figure 3.11: Number of users vs G_p for an $\frac{E_b}{N_0}$ = 10dB

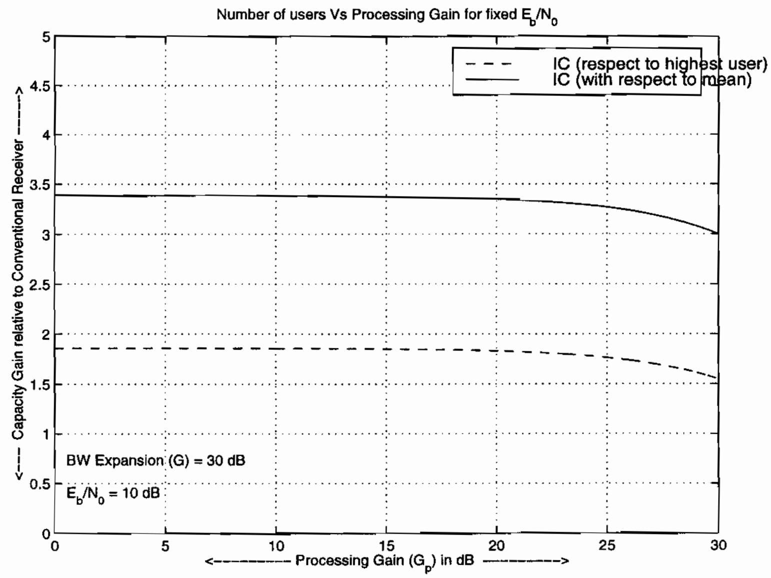


Figure 3.12: Capacity Gain vs G_p for an $\frac{E_b}{N_0}$ = 10dB

out-of-cell interference be λ; thus the total interference now increases by a factor of λ.

3.7.1 Performance of a Conventional Receiver

The rate achieved by any user in a multiple-cell system is from Equation 3.16,

$$R_d \leq \frac{W}{G_p}\log_2\left(1+\frac{\frac{E_b R_d G_p}{N_0}}{1+(1+\lambda)(K-1)\frac{E_b R_d}{N_0 W}}\right) \tag{3.37}$$

and from Equation 3.18, the number of users,

$$K \leq 1+\frac{G}{1+\lambda}\left[\frac{1}{\gamma_{req}}-\frac{1}{\frac{E_b}{N_0}}\right] \tag{3.38}$$

where,

$$\gamma_{req} = \left(\frac{2^{\frac{R_d G_p}{W}}-1}{\frac{G_p}{G}}\right) \tag{3.39}$$

The number of users in a multiple-cell system is similar to Equation 3.32, except it is decreased by a factor $(1+\lambda)$.

$$
\begin{aligned}
K &\leq 1+\frac{G}{(1+\lambda)}\left(1-\frac{1}{\frac{E_b}{N_0}}\right) && \text{when } G_p = G \\
&\leq 1+\frac{G}{(1+\lambda)}\left(\frac{1}{\ln 2}-\frac{1}{\frac{E_b}{N_0}}\right) && \text{when } G_p = 1 \text{ and } G >> 1
\end{aligned}
$$

Similar to the case of a single cell, for large G and large $\frac{E_b}{N_0}$, the capacity gain due to error control coding is 44 % compared to no error control coding. The capacity when compared to a single cell is reduced by a factor of $(1+\lambda)$.

3.7.2 Multiple cell System with SIC Receiver

The interference cancellation scheme in a multiple cell system is as shown in Fig. 3.13. The analysis for a SIC receiver operating in a multiple cell system is carried out under

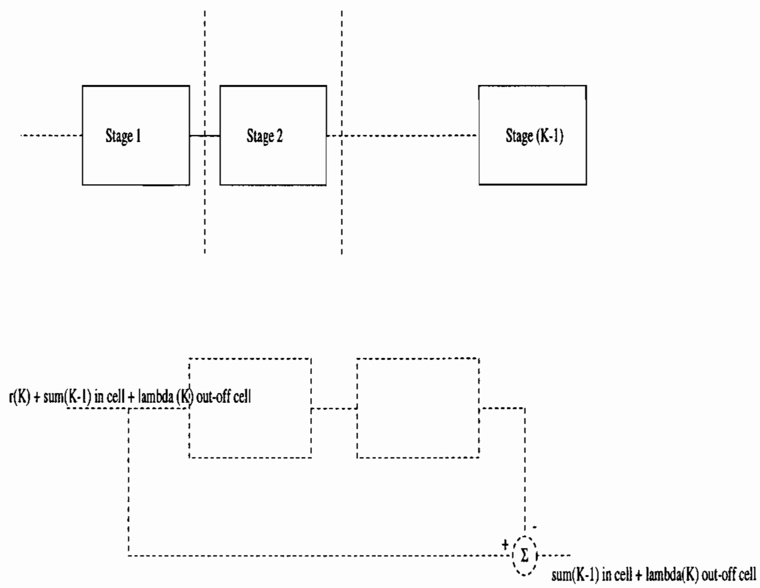


Figure 3.13: Successive Interference Cancellation Scheme

the assumption that the SIC receiver has knowledge of parameters of only in-cell users. At each stage, the in-cell interference is cancelled, and the out-of cell interference is left un-cancelled. Hence, the out of cell interference propagates to the last stage. Reasoning as above, the rate achieved by users with the ordering of the powers $P_1 < P_2 \ldots < P_K$, is

$$
\begin{aligned}
R_K &\leq \frac{W}{G_p}\log_2\left(1+\frac{\gamma_K G_p}{1+\sum_{i=1}^{K-1}\gamma_i+\lambda\sum_{i=1}^{K}\gamma_i}\right)\\
&\vdots\\
R_2 &\leq \frac{W}{G_p}\log_2\left(1+\frac{\gamma_2 G_p}{1+\gamma_1+\lambda\sum_{i=1}^{K}\gamma_i}\right)\\
R_1 &\leq \frac{W}{G_p}\log_2\left(1+\frac{\gamma_1 G_p}{1+\lambda\sum_{i=1}^{K}\gamma_i}\right)
\end{aligned}
\tag{3.40}
$$

where γ_i is the signal to noise power ratio, and λ is the out-of cell interference. Considering equal-rate case, $R_1 = R_2 = \ldots = R_K = R_d$, results in,

$$
\begin{aligned}
\gamma_2 &= \gamma_1\left[1+\frac{\gamma_1}{1+\lambda\sum_{i=1}^{K}\gamma_i}\right]\\
&\vdots\\
\gamma_K &= \gamma_1\left[1+\frac{\gamma_1}{1+\lambda\sum_{i=1}^{K}\gamma_i}\right]^{K-1}
\end{aligned}
\tag{3.41}
$$

from Equation 3.40,

$$
\frac{\gamma_1}{1+\lambda\sum_{i=1}^{K}\gamma_i} = \frac{\left[2^{\frac{R_d G_p}{W}}-1\right]}{G_p}
\tag{3.42}
$$

Let $M = \frac{2^{\frac{R_d G_p}{W}}-1}{\frac{G_p}{G}}$, substituting, this in Equation 3.40,

$$
\begin{aligned}
\gamma_2 &= \gamma_1\left[1+\frac{M}{G}\right]\\
&\vdots\\
\gamma_K &= \gamma_1\left[1+\frac{M}{G}\right]^{(K-1)}
\end{aligned}
\tag{3.43}
$$

Thus, even in the case of a multiple-cell system, the signal powers are in geometric series. The intuitive reasoning being that in the multiple-cell case all the users are affected by the out-of cell interference and thus all the signal powers need to be increased by an equal amount and thus the geometric series is maintained.

To obtain the number of users a SIC receiver can support; consider Equation 3.42,

$$\begin{aligned}
\frac{\gamma_1}{1+\lambda\sum_{i=1}^{K}\gamma_i} &= \frac{\left[2^{\frac{R_d G_p}{W}}-1\right]}{G_p} \\
\frac{\gamma_1}{1+\lambda\sum_{i=1}^{K}\gamma_i} &= \frac{M}{G}
\end{aligned} \tag{3.44}$$

since, the signal powers are in geometric series, the sum of the series can be computed; and is,

$$\sum_{i=1}^{K}\gamma_i = \frac{\gamma_1\left[\left(1+\frac{M}{G}\right)^K-1\right]}{\left(\frac{M}{G}\right)} \tag{3.45}$$

substituting, Equation 3.45, in Equation 3.44,

$$\begin{aligned}
\gamma_1 &= \left(\frac{M}{G}\right)\left\{1+\lambda\frac{\gamma_1\left[\left(1+\frac{M}{G}\right)^K-1\right]}{\left(\frac{M}{G}\right)}\right\} \\
\gamma_1 &= \frac{M}{G}\frac{1}{\left[1-\lambda\left(\left(1+\frac{M}{G}\right)^K-1\right)\right]} \\
\left(\frac{E_b}{N_0}\right)_1 &\le \frac{M}{\left[1-\lambda\left(\left(1+\frac{M}{G}\right)^K-1\right)\right]}
\end{aligned} \tag{3.46}$$

$$K \le \frac{\log\left(1+\frac{1}{\lambda}\left(1-\frac{M}{\left(\frac{E_b}{N_0}\right)_1}\right)\right)}{\log\left(1+\frac{M}{G}\right)} \tag{3.47}$$

The number of users a CDMA multiple cell system employing an SIC receiver can support is bounded by the out-of-cell interference and is inversely proportional to $\log(\lambda)$. The user capacities when a conventional receiver is employed in the system and a SIC receiver is employed in the system are compared in Figures. 3.14 and 3.15. The number of users supported by both the receivers is plotted as a function of $\frac{E_b}{N_0}$, for $G_p = 1$ and $G_p = G$. The capacity gain obtained by the use of an interference canceler is shown in Figures. 3.16 and 3.17 as a function of $\frac{E_b}{N_0}$. The variation of the capacity as a function of the processing gain G_p, is shown in Fig. 3.18 and the capacity gain as a function of processing gain Fig. 3.19.

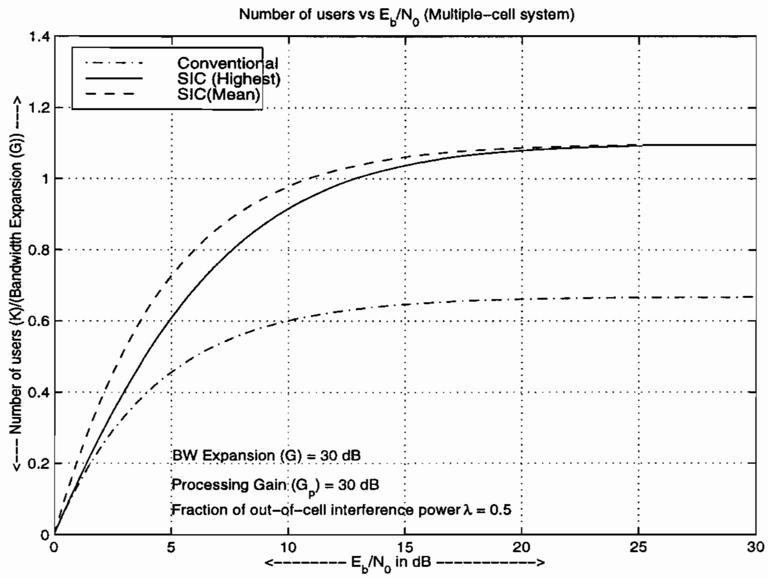


Figure 3.14: Number of users vs $\frac{E_b}{N_0}$, for $G_p = G$

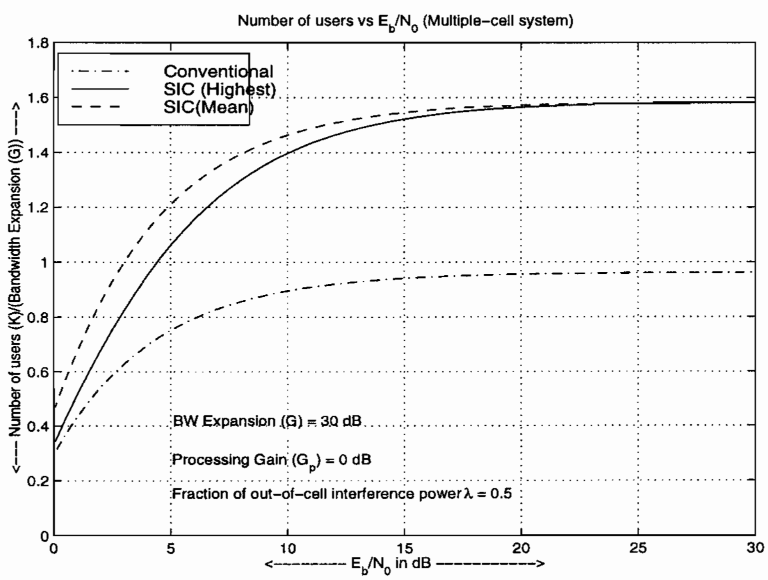


Figure 3.15: Number of users vs $\frac{E_b}{N_0}$, for $G_p = 1$

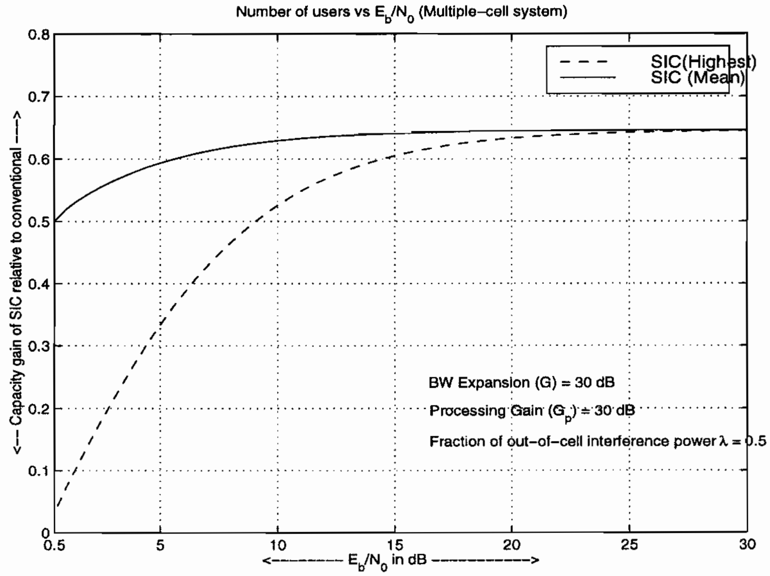


Figure 3.16: Capacity Gain vs $\frac{E_b}{N_0}$ for $G_p = G$

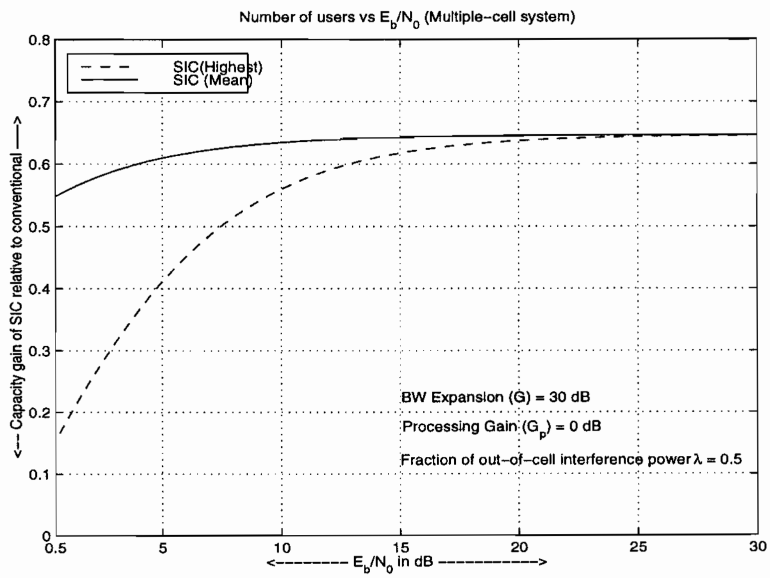


Figure 3.17: Capacity Gain vs $\frac{E_b}{N_0}$, for $G_p = 1$

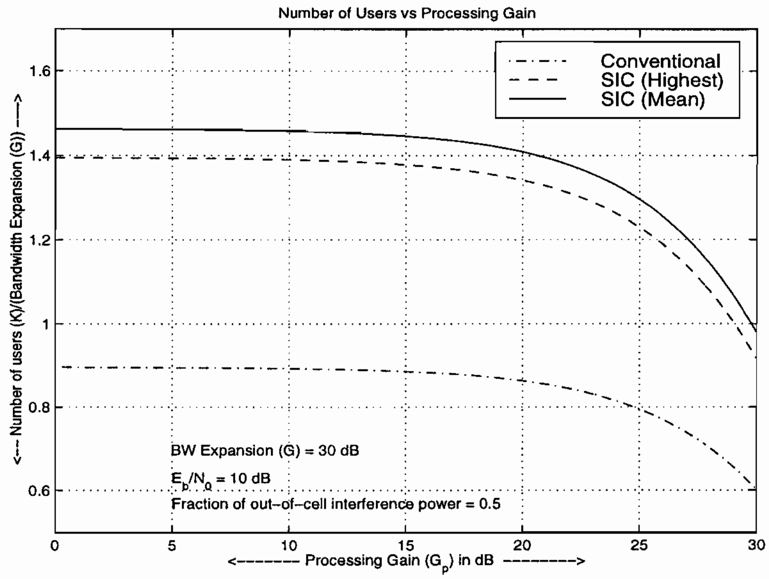


Figure 3.18: Number of users vs G_p for $\frac{E_b}{N_0}$ = 10 dB

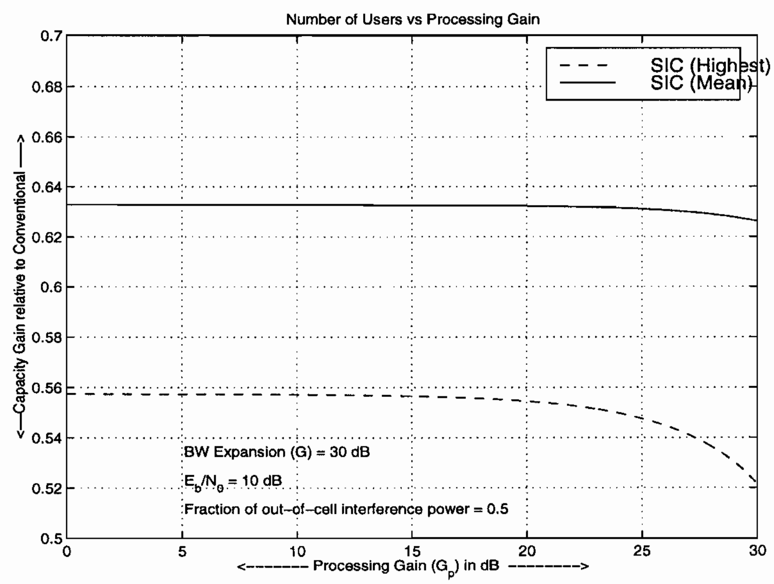


Figure 3.19: Capacity Gain vs G_p for $\frac{E_b}{N_0}$ = 10 dB

For a receiver used in a CDMA system where the bandwidth expansion is completely due to error control coding the error control decoder determines the order of complexity of the receiver. For an IC receiver without error control coding, the cancellation algorithm determines the order of complexity. Hence, from the point of complexity, it is interesting to consider the capacity gain that is realized when any IC scheme without error control coding is compared to a conventional receiver where bandwidth expansion is completely due to error control coding. Fig. 3.20 plots the capacity gains of optimal IC and SIC relative to a conventional receiver with and without error control coding. The gain of a SIC receiver at $\lambda = 0.5$ when compared to a conventional receiver without error coding is around 65% and when compared to a conventional receiver with error control coding is 14%. These results suggest that the complexity and the gain of the IC scheme should be considered before implementing an IC scheme. After implementation, if it turns out that the complexity of IC scheme is very much greater than an error control decoder, then the gain provided by the IC scheme should be considered before implementing the receiver. The gain that an ideal IC provides

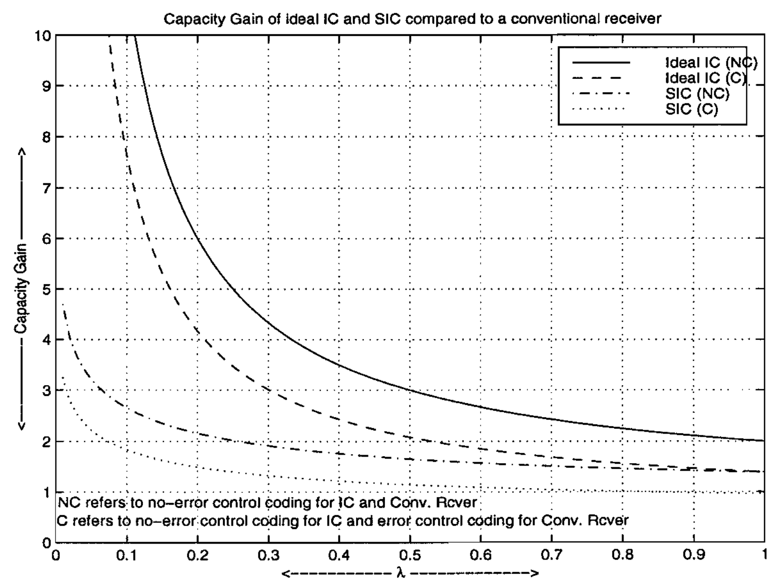


Figure 3.20: Gains of ideal IC and ideal SIC relative to conventional receiver

over a SIC receiver as a function of λ is shown in Fig. 3.21. The figure shows that the performance of an ideal SIC receiver is considerably less than an optimal IC. For

$\lambda = 0.5$, the gain of an optimal IC is around 80% more than an ideal SIC. This result suggests that for the specific power distribution which was considered, cancelling interference using a successive cancellation mechanism may not be the best possible way to realize the theoretical maximum capacity of a CDMA system.

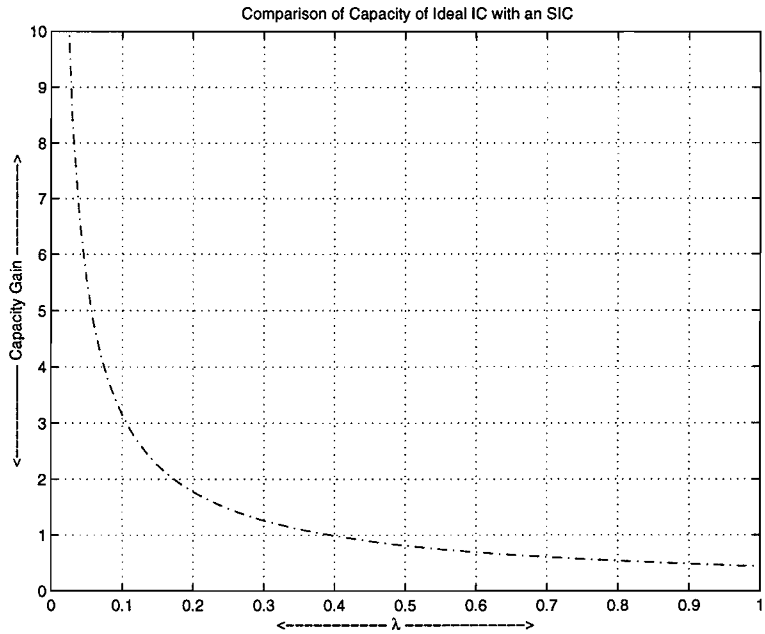


Figure 3.21: Capacity gain of ideal IC relative to SIC

3.8 Summary

The results in this chapter indicate that to maximize the user capacity; bandwidth expansion should be allocated to error control coding. This conclusion is independent of the receiver being employed in CDMA system. These results are similar to the results proposed by Hui [38], Viterbi [29] and others. Interestingly, the results suggest that the average capacity of a CDMA system is independent of the channel characteristics when $G_p = 1$.

An information theoretic approach to evaluate the capacity of a CDMA was presented in this chapter. The idea was to develop an approach to calculate the user capacities of SIC and PIC receivers vis-a-vis a conventional receiver. The capacity of a

SIC receiver was obtained using this approach and the approach was not suitable to evaluate the capacity of a PIC receiver. Hence, the performance of SIC receiver was compared to the optimal interference cancellation receiver. The results from the comparison suggested that for the specific power distribution that was considered here, a SIC receiver structure might not be the best possible approach for interference cancellation. As the motivation for this technical report was to develop an algorithm based on a receiver structure which provides a larger user capacity the conclusion that SIC receiver structure may not be the best possible scheme for interference cancellation prompted us to consider a PIC scheme as the basic receiver structure for developing a new receiver algorithm. In the following chapters, the concept of PIC is presented and followed by the chapter where modifications to the basic PIC scheme are presented.

Chapter 4

Concept of Parallel Interference Cancellation

Parallel interference cancellation scheme is a non-linear interference cancellation technique which cancels MAI utilizing the knowledge of estimates of other users. The general receiver structure consists of a number of stages; each stage refining the estimates of previous stages. Various PIC algorithms for DS-CDMA systems have been proposed [28, 1, 50] and the performance of these receivers is shown to be considerably better than the conventional receiver. Two of the proposed algorithms are presented in this chapter. The multi-stage PIC receiver proposed by Kaul and Woerner[50] and the improved parallel cancellation receiver proposed by Divsalar *et al* [1] are the receiver structures that are presented in this chapter. The modified PIC algorithm proposed in the next chapter combines both these approaches.

4.1 Preliminaries

The concept of parallel IC scheme is discussed in this section. Let the received signal be r(t),

$$r(t) = \sum_{i=1}^{K} s_i(t) + n(t) \tag{4.1}$$

$$= \sum_{i=1}^{K} \sqrt{P_i} b_i(t) c_i(t) e^{j\phi_i} + n(t) \tag{4.2}$$

where, P_i, b_i, c_i and ϕ_i refers to the power, bit, PN code and phase of user i. As already stated, the optimal multiuser detector [14] is derived from a joint ML detection of the K user bits. Thus the receiver has a complexity which is exponential in the number of users. To reduce the complexity of the algorithm, a single-shot detector is assumed; all the data bits are estimated by observing a single bit interval. To implement such an algorithm, theoretically knowledge of all the other users data bits is required. Since, knowledge of other users data bits is not known, the estimates of the other users data bits are used. This approach suggests an iterative scheme wherein each stage produces new and better estimates of the user bits. Hence, the decision rule for user i's bit at the mth iteration stage is,

$$\widehat{b}_i(m) = sgn\left(y_i(m) - \sum_{j=1, j\neq i}^{K} \widehat{s}_j(m-1) \right) \tag{4.3}$$

where $\widehat{s}_j(m-1)$ are estimates derived from the previous stage estimates and $y_i(m)$ is the correlator output at stage m. Hence, the idea is to process the received signal simultaneously followed by estimation of MAI. Subtracting the MAI estimates from the estimates of the current stage produces new and better decision estimates. A multi-stage iterative approach for parallel interference cancellation was proposed by Varanasi and Aazhang [28] and this idea was used in the receivers proposed by Kohono and Imai [13]. At each stage of the receiver, the estimated interference was cancelled completely as in Equation 4.3. Divsalar *et al.* showed that the idea of total interference cancellation is not the best way to cancel interference [1]. They proposed an improved parallel cancellation scheme; instead of cancelling the estimated MAI completely they weigh the previous stage estimates and this weighted estimate is then cancelled. Another multi-stage PIC receiver was proposed by Kaul and Woerner [50]; instead of using the hard decisions of the previous stage, the soft decision estimates are re-spread and subtracted from the received signal. It was subsequently shown that for the multi-stage PIC receiver, the estimates from second-stage are biased and Bueherer *et al.* [51] proposed a bias reduction technique. Recently, Renucci [52], optimized the partial cancellation factors. The optimal partial cancellation factors are dependent on the number of users in the system, their powers and the thermal noise variance.

Our proposed modification combines the approaches of Divsalar *et al.* [1] and Bueherer *et al* [51] and is similar to the approach of Renucci [52]. In section 4.2, performance analysis of a multistage PIC proposed by Kaul *et al.* [50] is discussed. The reasons for bias in the estimates of multistage PIC are discussed and is followed by the modifications proposed by Correal *et al.* In section 4.3, the performance of an improved parallel interference cancellation scheme proposed by Divsalar *et al* is analyzed. These two PIC receivers are presented in this chapter since the proposed modification is based on these two algorithms.

4.2 Multistage Parallel Interference Cancellation Receiver

The block diagram of a multi-stage PIC receiver is shown in Fig. 4.1. Each stage consists of a bank of correlators. The received signal r(t) is,

$$r(t) = \sum_{i=1}^{K} s_i(t-\tau_k) + n(t) \tag{4.4}$$

where, s(t) is given as,

$$s_i(t) = \sqrt{2P_i}b_i(t)c_i(t)\cos(\omega_c t + \phi_i)$$

where, $b_i, c_i(t), \tau_i$,and ϕ_i denote the bit, PN code, delay and phase of user i respectively. The correlator outputs are given by [50] (assuming that the receiver has perfect

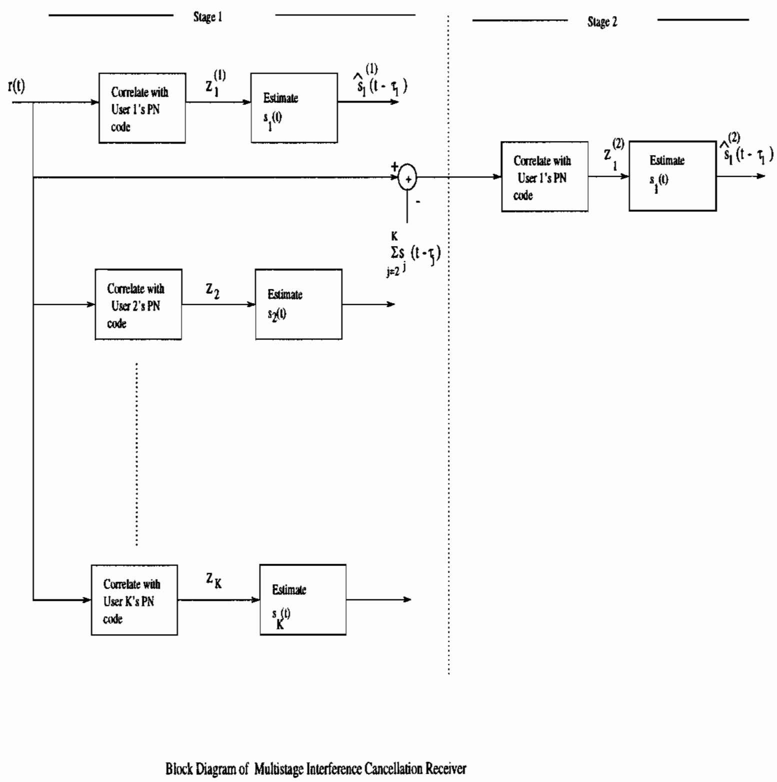


Figure 4.1: Block Diagram of Multistage PIC

knowledge of phase and timing information),

$$\begin{aligned} Z_i^{(1)} &= \int_{t=(j-1)T+\tau_i}^{jT+\tau_i} r(t)c_i(t-\tau_i)\cos(\omega_c t+\tau_i) \\ &= \sqrt{\frac{P_i}{2}}b_i T + \sum_{k=1,k\neq i}^{K} \sqrt{\frac{P_k}{2}}b_k\cos(\phi_k-\phi_i)T + \chi_i \\ &= D_i + I_i^{(1)} + \chi_i \end{aligned} \tag{4.5}$$

where D_i is the desired user's estimate, $I_i^{(1)}$ is the interference due to presence of other users, and χ_i is the output due to noise. The correlator output consists of three terms, the desired signal, the MAI term and noise term. The receiver's task is to cancel the multiple access term that is present at the correlator output. The correlator estimates are multiplied by the respective PN codes to obtain the re-spread estimates. These re-spread estimates of other users are subtracted from the received signal to provide a new signal estimate to be passed onto the next stage. The re-spread estimate $\hat{s}_i^{(1)}(t)$ is,

$$\hat{s}_i^{(m)}(t) = \frac{2}{T}Z_i^{(m)}(t)c_i(t)\cos(\omega_c t+\phi_i) \tag{4.6}$$

The superscript refers to number of stage and subscript refers to the user number. The soft decision estimate of mth stage is $Z_i^{(m)}$. The desired user's new signal estimate that is passed to the subsequent stage is,

$$\hat{s}_i^{(m+1)}(t) = r(t) - \sum_{k=1,k\neq i} \hat{s}_k^{(m)}(t-\tau_k) \tag{4.7}$$

The new signal estimate is passed to the correlator to form a new refined estimate. For the multi-stage PIC, the BER is [50],

$$P_b^{(m)} = Q\left(\sqrt{\frac{1}{\frac{1}{2\frac{E_b}{N_0}}\left[\frac{1-(\frac{K-1}{3N})^m}{1-(\frac{K-1}{3N})}\right] + (\frac{K-1}{3N})^m}}\right) \tag{4.8}$$

The BER equation is obtained by assuming that the estimates of the correlator at any stage are un biased [50]. It is shown later [53] that the assumption that the estimates at any stage are unbiased is not correct. The assumption that the estimates are unbiased predicts an optimistic BER than that can be practically obtained. The variation of bias with respect to number of users is discussed in the next section.

4.2.1 Analysis of bias in the estimates of a multi-stage PIC

A synchronous system is assumed to analyze the bias in the estimates. The problem of bias in the estimates was discussed in [53]. The estimated reconstructed signal for user i at stage m is given by,

$$\hat{s}^{(m)}(t) = \frac{2}{T}c_j(t)\cos(\omega_c t + \phi_j)\sum_{i=-\infty}^{\infty} Z_{j,i}^{(m)} p_T(t - iT) \tag{4.9}$$

where $P_T(t)$, is a unit pulse function of duration T equal to the bit period. For ease of analysis, assume that there are only two users in the system. The decision statistic at stage 1, for bit 1 of user 1 is,

$$Z_{1,i}^{(1)} = T\sqrt{\frac{P_1}{2}}b_{1,i} + \sqrt{\frac{P_2}{2}}\cos(\phi_2 - \phi_1)b_{2,i}\rho_{21} + \chi_1 \tag{4.10}$$

where ρ_{12} is the cross-correlation between codes of user 1 and 2, χ_i is the output due to noise. Similarly, for user 2 the decision statistic is,

$$Z_{2,i}^{(1)} = T\sqrt{\frac{P_2}{2}}b_{2,i} + \sqrt{\frac{P_1}{2}}\cos(\phi_1 - \phi_2)b_{1,i}\rho_{12} + \chi_2 \tag{4.11}$$

These estimates are re-spread and subtracted from the received signal to form estimates as described in Equation 4.9. These estimates are then passed to the next stage. The decision statistic for user 1 at stage 2 is,

$$Z_{1,i}^{(2)} = Z_{1,i}^{(1)} - \cos(\phi_2 - \phi_1)Z_{2,i}^{(1)}\rho_{21}\rho_{21} \tag{4.12}$$

substituting, Equation 4.10, Equation 4.11 in Equation 4.12 and including only the terms which contain P_1 and b_1,

$$Z_{1,i}^{(2)} = T\sqrt{\frac{P_1}{2}}b_1 - \frac{\cos^2(\phi_2 - \phi_1)}{T}\frac{P_1}{2}\rho_{21}\rho_{21}b_1 \tag{4.13}$$

It is noticed that due to the process of cancellation, there are other terms which contain P_1 and b_1. This leads to the bias in the estimates. Taking the expected value of Equation 4.13,

$$E\{Z_{1,i}^{(2)}/b_{1,i}\} = T\frac{P_1}{2}b_{1,i} - \frac{P_1}{2}E\{\frac{\cos^2(\phi_2 - \phi_1)}{T}\frac{P_1}{2}\rho_{21}\rho_{21}b_1\} \tag{4.14}$$

If MAI is approximated to be Gaussian then the variance of $E\{\rho_{21}^2\}$ is $\frac{2N}{3}$ [19], and substituting this result in Equation 4.14 results in,

$$E\{Z_{1,i}^{(2)}/b_{1,i}\} = N\frac{P_1}{2}b_{1,i}[1 - \frac{2}{3N}] \tag{4.15}$$

and for a K user system, the bias is given by,

$$E\{Z_{1,i}^{(2)}/b_{1,i}\} = N\frac{P_1}{2}b_{1,i}[1 - \frac{2(K-1)}{3N}] \tag{4.16}$$

The bias in the estimates increases with system loading. Imperfect interference cancellation not only affects the mean of the decision statistics, but also it affects the variance. Hence instead of cancelling the interference completely, partial parallel interference cancellation was used by Bueherer *et al.* The idea of partial interference cancellation was first proposed in [1]. Instead of cancelling the estimated interference completely, the previous stage estimates are multiplied by a factor δ $(0 < \delta < 1)$. The new estimates that are passed to the subsequent stage are,

$$\hat{s}_k^{(m+1)}(t) = r(t) - \delta_k^{(m)} \sum_{i \neq k} \hat{s}_i^{m}(t) \tag{4.17}$$

The value of δ determines the amount of interference that is cancelled. The BER after m stages of cancellation will depend on the partial cancellation factors δ at each stage. In [53], the partial cancellation factor was assumed to be constant and the value chosen was 0.7 for $\delta^{(1)}$. In this technical report, the partial cancellation factors δ are chosen in order to minimize the variance of total interference. A somewhat similar approach was considered by Renucci [52]. The parameter which is minimized by Renucci is the BER after the first stage of cancellation. The minimization was performed over all the users. It turns out that the partial cancellation factors are dependent on the number of users, powers of the users and the noise variance. For perfect power control case, the optimum partial cancellation factors are

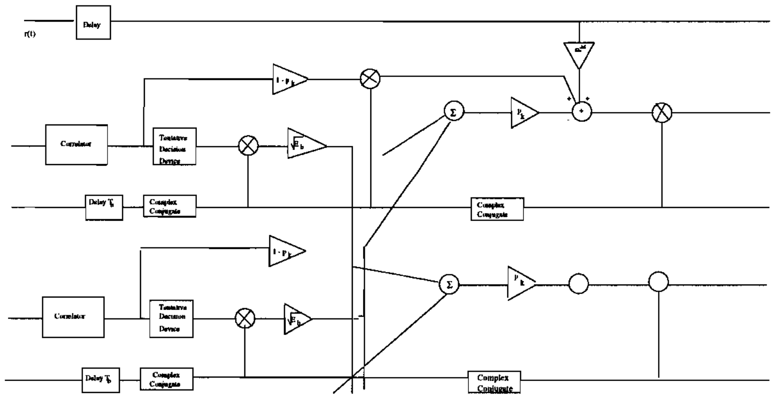
$$\delta = \frac{4PN^2 - 2PN - 4N^2 var(\chi)}{4PN^2 + 2PK(2N-1) - 3PN - 2P + 4N^2 var(\chi)} \tag{4.18}$$

4.3 Improved Parallel Cancellation Receiver

The idea of partial parallel interference cancellation was first proposed by Divsalar, Simon and Raphaeli [1]. The block diagram of the iterative parallel interference cancellation receiver is shown in Fig 4.2. The argument for proposing partial cancellation is as follows; as the estimates of initial stages are not reliable, decisions based on these estimates tend to be unreliable. Thus, total cancellation of the estimated multiuser interference will not lead to the best decision metric. They presented an iterative algorithm, where, in the early stages, only a fraction of the multiuser interference is cancelled with the amount of interference cancellation increasing as the number of stages increase. Their idea was to form a weighted sum from considerations based on jointly observing the current correlator output and the decisions at the previous stage. The algorithm hence is,

$$\tilde{b}_1(k) = p_k Z_1^{(k)} + (1 - p_k)\tilde{b}_1(k-1) \tag{4.19}$$

where, $\tilde{b}_1(k)$ is the soft decision of the current stage and $\tilde{b}_1(k-1)$ are soft decision estimates of the previous stage. The value p_k lies between 0 and 1 and is the partial cancellation factor. The values of p_k, were found from simulations. In summary, the improved parallel cancellation approach uses partial parallel interference cancellation and at each stage utilizes the estimates of the previous stage as well as the present stage, followed by a non-linear decision device. They considered a hyperbolic tangent decision device to obtain the soft decision estimates of the previous stage. Although the algorithm is straightforward, a complex approach was presented to implement the algorithm. Such a receiver with non-linear estimation was shown to provide a gain of around 9, at a BER of 10^{-2}.



Stage of improved parallel cancellation

Figure 4.2: Improved Parallel Cancellation

4.4 Chapter Summary

A review of the earlier work in the area of parallel cancellation was presented. Two receiver algorithms are discussed; both the receiver algorithms use partial parallel cancellation instead of brute force cancellation. The algorithm presented by Divsalar *et al.* is simple but the approach presented is very complex. The multi-stage PIC with partial cancellation factors is simple to implement but does not support a large number of users as supported by improved PIC algorithm. A new modified PIC algorithm is presented in the next chapter.

Chapter 5

A Modified Parallel Interference Cancellation Receiver

In this chapter a modified parallel interference cancellation algorithm is presented. The proposed algorithm incorporates the idea of partial parallel interference cancellation proposed by Divsalar *et al.* [1]. In the partial parallel interference cancellation algorithm proposed by Divsalar *et al.* [1], the estimates of the previous stage are weighted by a number between [0,1], and this factor is called the partial cancellation factor. In their algorithm, the partial cancellation factors are constant for any number of users in the system and the cancellation factors which reduce the BER are calculated from simulations. In the proposed modification, the partial cancellation factors are calculated to minimize the total interference variance after a single stage of cancellation. It is shown that the calculated partial cancellation factors are dependent on the number of users in the system, the processing gain and the received signal powers of users. The receiver has knowledge of the number of users in the system, the processing gain and not the received signal powers, hence, there is a necessity to estimate the received signal powers in order to calculate the partial cancellation factors. In fact, to overcome this problem, a method to estimate the received signal powers as part of the cancellation procedure is presented. Apart from knowledge of number of users and processing gain, the receiver is assumed to have knowledge of phase and delay parameters.

The modified algorithm for interference cancellation along with the method to estimate the received signal powers is presented in 5.1 . The complex base-band simulation model developed to simulate the system is presented in 5.2. Analytical and simulation results as well as comparisons with the already proposed receivers are presented in section 5.3. The complexity of the receiver is presented in terms of the number of operations required per bit decision in section 5.4.

5.1 Modified Parallel Interference Cancellation Algorithm for a synchronous DS-CDMA system

The modified PIC algorithm along with the method to estimate the received signal powers is presented in this section. An argument that a synchronous CDMA system is the worst-case system with respect to the MAI suffered by the correlator estimates. Hence, throughout the analysis we consider a synchronous CDMA system and also present the changes that need to be done to the algorithm when it is implemented for an asynchronous system. The system model is similar to the model presented in chapter 2, except a synchronous system model is assumed. It is assumed that there are K users in the system transmitting at the same data rate. The received signal at the base station is the sum of the transmitted signals and is,

$$r(t) = \sum_{i=1}^{K} s_i(t) + n(t) \tag{5.1}$$

where n(t) is Gaussian noise with a variance $N_0/2$, s_i is the transmitted signal of user i,

$$s_i(t) = \sqrt{2P_i}b_i(t)c_i(t)\cos(\omega_c t + \phi_i) \tag{5.2}$$

where P_i, c_i, b_i and ϕ_i are the received signal power, spreading sequence, transmitted bit and phase of user i. The front-end of the receiver down-converts the received signal to base-band. The received base-band signal is then,

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k}b_k(t)c_k(t)\cos(\omega_c t + \phi_k) + n(t)$$

Each stage of the receiver consists of a bank of correlators. The correlator output for bit 0 of user i is,

$$
\begin{aligned}
Z_{i,0}^{(1)} &= \frac{1}{T}\int_{t=0}^{t=T} r(t)c_1(t)\cos(\omega_c t+\phi_i)dt \qquad (5.3)\\
&= \frac{1}{T}\int_{t=0}^{t=T}\left(\sum_{k=1}^{K}\sqrt{2P_k}b_k(t)c_k(t)\cos(\omega_c t+\phi_k)+n(t)\right)c_i(t)\cos(\omega_c t+\phi_i)dt\\
&= \begin{cases}
\frac{1}{T}\int_{t=0}^{t=T}\sqrt{2P_i}b_i(t)c_i(t)\cos(\omega_c t+\phi_1)c_i(t)\cos(\omega_c t+\phi_i)dt+\\
\frac{1}{T}\int_{t=0}^{t=T}\left(\sum_{k=1,k\neq i}^{k=K}\sqrt{2P_k}b_k(t)c_k(t)\cos(w_c t+\phi_k)\right)c_i(t)\cos(\omega_c t+\phi_i)dt+\\
\frac{1}{T}\int_{t=0}^{T} n(t)\cos(\omega_c t+\phi_i)dt
\end{cases}\\
&= \sqrt{\frac{P_i}{2}}b_{i,0}+I_{i,0}^{(1)}+\chi_{i,0}
\end{aligned}
$$

The correlator detector is a single-shot detector and hence the output statistics are similar for all bits and is independent of the time index,

$$
Z_i^{(1)} = \sqrt{\frac{P_i}{2}}b_i+I_i^{(1)}+\chi_i \qquad (5.4)
$$

where, $Z_i^{(1)}$, represents the correlator output of user i after the first stage, $I_i^{(1)}$ represents the MAI interference to the correlator output of user i and χ_i is the noise at the output of the correlator output. Here, the superscript refers to the stage and the subscript refers to the user.

$$
\begin{aligned}
I_i^{(1)} &= \frac{1}{T}\sum_{k=1,k\neq i}^{K}\sqrt{2P_k}b_k\int_{t=0}^{T} c_k(t)\cos(\omega_c t+\phi_k)c_1(t)\cos(\omega_c t+\phi_i)dt\\
&= \sum_{k=1,k\neq i}^{K}\sqrt{\frac{P_k}{2}}b_k\rho_{ki}\cos(\phi_k-\phi_i) \qquad (5.5)
\end{aligned}
$$

$$
\chi_i = \frac{1}{T}\int_{t=0}^{T} n(t)c_i(t)\cos(\omega_c t+\phi_i)dt \qquad (5.6)
$$

where,

$$
\frac{1}{T}\int_{t=0}^{T} c_i(t)c_j(t)dt = \begin{cases} 1 & \text{if } i=j\\ \rho_{ij} & \text{if } i\neq j\end{cases} \qquad (5.7)
$$

The first stage correlator bank outputs are,

$$Z_1^{(1)} = \sqrt{\frac{P_1}{2}} b_1 + \sum_{i=2}^{K} \rho_{i1} \sqrt{\frac{P_i}{2}} b_i \cos(\phi_i - \phi_1) + \chi_1 \tag{5.8}$$

$$Z_2^{(1)} = \sqrt{\frac{P_2}{2}} b_2 + \sum_{i=1, i \neq 2}^{K} \rho_{i2} \sqrt{\frac{P_i}{2}} b_i \cos(\phi_i - \phi_2) + \chi_2 \tag{5.9}$$

$$\vdots$$

$$Z_K^{(1)} = \sqrt{\frac{P_K}{2}} b_K + \sum_{i=1}^{K-1} \rho_{iK} \sqrt{\frac{P_i}{2}} b_i \cos(\phi_i - \phi_K) + \chi_K$$

As discussed earlier, the concept of parallel interference cancellation is to use the correlator outputs and estimate the MAI to the correlator outputs. A method is to re-spread the soft correlator outputs and form a new re-spread estimate that is passed to the subsequent stage. This is the method that is implemented in multi-stage PIC. The re-spread estimates are,

$$\hat{s}_i^{(1)}(t) = \frac{2}{T} Z_i^{(1)} \cos(\omega_c t + \phi_i) c_i(t) \tag{5.10}$$

and the new estimate of the received signal for each user that is passed to the subsequent stage is,

$$s_i^{(2)} = r(t) - \sum_{k=1, k \neq i}^{K} \frac{2}{T} Z_k^{(1)} \cos(\omega_c t + \phi_k) c_k(t) \tag{5.11}$$

This new received signals are then passed to the subsequent correlator bank. Instead of subtracting the re-spread estimates from the received signal to form $s_i^{(2)}$, the re-spread estimates are used to calculate the estimates of interference $I_i^{(1)}$ that effect the first correlator bank decision estimates. The interference estimate for user i is calculated as follows,

$$\begin{aligned} \hat{I}_i^{(1)} &= \frac{1}{T} \int_{t=0}^{T} \sum_{k=1, k \neq i}^{K} \hat{s}_k^{(1)}(t) c_i(t) \cos(\omega_c t + \phi_i) dt \qquad (5.12) \\ &= \frac{1}{T} \int_{t=0}^{T} \left(\sum_{k=1, k \neq i}^{K} \frac{2}{T} Z_k^{(1)} \cos(\omega_c t + \phi_k) c_k(t) \right) c_i(t) \cos(\omega_c t + \phi_i) dt \end{aligned}$$

substituting for Z's from (5.3), in (5.13), we obtain,

$$
\begin{aligned}
\widehat{I}_i^{(1)} &= \frac{1}{T}\int_{t=0}^{T}\left(\sum_{k=1,k\neq i}^{K}\frac{2}{T}\left[\sqrt{\frac{P_k}{2}}b_kT + I_k^{(1)} + \chi_k\right]c_k(t)\cos(\omega_c t+\phi_k)\right)c_i(t)\cos(\omega_c t+\phi_i)dt \\
\widehat{I}_i^{(1)} &= \begin{cases} \frac{1}{T}\sum_{k=1,k\neq i}^{K}\int_{t=0}^{T}\sqrt{2P_k}b_kc_k(t)\cos(\omega_c t+\phi_k)c_i(t)\cos(\omega_c t+\phi_i)dt \\ \frac{2}{T}\sum_{k=1,k\neq i}^{K}\int_{t=0}^{T}I_k^{(1)}c_k(t)\cos(\omega_c t+\phi_k)c_i(t)\cos(\omega_c t+\phi_i)dt \\ +\frac{2}{T}\sum_{k=1,k\neq i}^{K}\int_{t=0}^{T}\chi_kc_k(t)\cos(\omega_c t+\phi_k)c_i(t)\cos(\omega_c t+\phi_i)dt \end{cases} \\
\widehat{I}_i^{(1)} &= I_i^{(1)} + I_{uw,i}^{(1)} + \chi_{p,i} \qquad (5.13)
\end{aligned}
$$

The estimated interference $\widehat{I}_i^{(1)}$ is subtracted from $Z_i^{(1)}$ to form a new modified estimate $Z_{mod,i}$ which is given as,

$$
\begin{aligned}
Z_{mod,i}^{(1)} &= Z_i^{(1)} - \widehat{I}_i \qquad (5.14)\\
&= D_i - I_{uw,i}^{(1)}
\end{aligned}
$$

From Equation 5.13 it can be noticed that the interference estimates similar to the correlator estimates have three terms, the desired interference estimate I_i, the unwanted interference $I_{uw,i}^{(1)}$ and a noise term $\chi_{p,i}$. The unwanted interference $I_{uw,i}^{(1)}$ is the result of using the unreliable correlator estimates of the first stage. Since, the calculated interference estimate Equation 5.13 consists of unwanted interference estimates, complete subtraction of interference estimates from the previous stage outputs results in unwanted interference estimates being passed to the subsequent stage. Instead of completely subtracting these calculated interference estimates $\widehat{I}_i^{(1)}$ from $Z_i^{(1)}$, in the method of partial parallel interference cancellation method, the interference estimates $\widehat{I}_i^{(1)}$ are weighted by a number between [0,1] and are then subtracted from from $Z_i^{(1)}$. The modified parallel cancellation algorithm also uses the concept of partial cancellation. Similar to the improved parallel cancellation algorithm, the previous stage correlator estimates and the modified correlator estimates are combined as follows,

$$\begin{aligned}
Z_{f1,i} &= \zeta(K)Z^{(1)}_{mod,i} + (1-\zeta(K))Z^{(1)}_i \qquad (5.15)\\
&= \zeta(K)\left(D_i + I^{(1)}_i + \chi_i - I^{(1)}_i + I^{(1)}_{uw,i} + \chi_{p,i}\right) + (1-\zeta(K))(D_i + I^{(1)}_i + \chi_i)\\
&= D_i + I^{(1)}_i(1-\zeta(K)) - \zeta(K)I^{(1)}_{uw,i} - \zeta(K)\chi_{p,i} + \chi_i\\
&= D_i + I_{total,i}
\end{aligned}$$

where D_i is the desired decision estimate, $I_{total,i}$ is the total interference which includes noise and MAI terms. In the improved parallel cancellation algorithm where the partial cancellation coefficients are independent of the number of users in the system. In the modified parallel cancellation algorithm, the coefficients ζ are calculated to minimize the variance of total interference $I_{total,i}$. We show later in this section that the partial cancellation coefficients ζ are dependent on the number of users in the system, processing gain and received signal powers of the users. It can be noticed from Equation 5.16 that the total interference consists of four terms: the interference estimate at the first stage correlator output, the unwanted interference estimate at the output of the modified correlator output, noise term at the correlator output and noise estimate at the modified correlator output. The variance of total interference is calculated in the next section.

5.1.1 Evaluation of variance of the total interference ($I_{total,i}$)

The variance of $I_{total,i}$ is,

$$\begin{aligned}
var(I^{(1)}_{total,i}) &= Var\left(I^{(1)}_i(1-\zeta) - \zeta I^{(1)}_{uw,i} - \zeta\chi_{p,i} + \chi_i\right) \qquad (5.16)\\
&= \left\{ \begin{array}{l} E\left[\left(I^{(1)}_i(1-\zeta) - \zeta I^{(1)}_{uw,i} - \zeta\chi_{p,i} + \chi_i\right)^2\right] - \\ \left[E\left(I^{(1)}_i(1-\zeta) - \zeta I^{(1)}_{uw,i} - \zeta\chi_{p,i} + \chi_i\right)\right]^2 \end{array}\right.
\end{aligned}$$

where E(x) is the expectation operation over random variable x and var(x) is variance of random variable x. Expanding Equation 5.17,

$$var(I^{(1)}_{total}) = \begin{cases} (1-\zeta)^2 E[(I_i^{(1)}).^2] + (\zeta)^2 E[(I^{(1)}_{uw,i})^2] + (\ zeta)^2 E[(\chi_{p,i})^2] + E[(\chi_i)^2] \\ -2(1-\zeta)\zeta E[I_i^{(1)} I^{(1)}_{uw,i}] - 2(1-\zeta)E[I_i^{(1)}\chi_{p,i}] + 2(1-\zeta)E[I_i^{(1)}\chi_i] \\ +2(\zeta)^2 E[I^{(1)}_{uw,i}\chi_{p,i}] - 2\zeta E[I^{(1)}_{uw,i}\chi_i] - 2\zeta E[\chi_i \chi_{p,i}] \\ -\left[E\left(I_i^{(1)}(1-\zeta) - \zeta I^{(1)}_{uw,i} - \zeta\chi_{p,i} + \chi_i\right)\right]^2 \end{cases}$$

The variance of each term is evaluated individually.

From Equation 5.13, the interference term is,

$$I_i^{(1)} = \sum_{k=1,k\neq i}^{K} \sqrt{\frac{P_k}{2}} b_k \cos(\phi_k - \phi_i)\rho_{ki} \tag{5.17}$$

Hence, the variance of $I_i^{(1)}$, is,

$$\begin{aligned} var(I^{(1)})_i) &= E\left\{\left(I_i^{(1)}\right)^2\right\} - \left(E[I_i^{(1)}]\right)^2 && (5.18) \\ &= E\left\{\left[\sum_{k=1,k\neq i}^{K} \sqrt{\frac{P_k}{2}} b_k \rho_{ki} \cos(\phi_k - \phi_i)\right]^2\right\} - \left(E[I_i^{(1)}]\right)^2 \end{aligned}$$

The variance of $I_i^{(1)}$ from [8, 19] is,

$$var(I_i^{(1)}) = \frac{\sum_{k=1,k\neq i}^{K} P_k}{2N} \tag{5.19}$$

The variances of $I^{(1)}_{uw,i}$ and the cross-correlations of $I^{(1)})_i$, $I^{(1)}_{uw,i}$ and $\chi_i, \chi_{p,i}$ are evaluated in appendix A. The evaluated variances are,

$$\begin{aligned} var(I^{(1)}_{uw,i}) &= \begin{cases} \frac{1}{4N^2}\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq k,i}^{K} P_j + \frac{1}{4N^2}(K-2)(K-3)P_i \\ +\frac{(K-3)}{8N^3}\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq i,k}^{K} P_j + \frac{3}{8}(K-1)P_1\left(\frac{3}{N^2} - \frac{2}{N^3}\right) \end{cases} \\ var(\chi_i) &= \frac{N_0}{2T} \\ var(\chi_{p,i}) &= \frac{(K-1)}{2N}\frac{N_0}{2T} \\ E\left[I_i^{(1)}, I^{(1)}_{uw,i}\right] &= \left(\frac{\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq i,k}^{K} P_j}{4N^2}\right) && (5.20) \end{aligned}$$

all the other cross-correlation terms turn out to be zero. Hence the total interference variance Equation 5.17 is,

$$var(I_{total}^{(1)}) = \begin{cases} (1-\zeta)^2\left[\frac{\sum_{k=1,k\neq i}^{K} P_k}{2N}\right] + \zeta^2\left[\frac{\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq k}^{K} P_j}{N^2}\right] \\ +\zeta^2\frac{1}{4N^2}(K-2)(K-3)P_i + \zeta^2\frac{(K-3)}{8N^3}\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq i,k}^{K} P_j \\ +\zeta^2\frac{3}{8}(K-1)P_1\left(\frac{3}{N^2}-\frac{2}{N^3}\right) \\ -\frac{2(1-\zeta)(\zeta)\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq i,k}^{K} P_j}{4N^2} \\ +\frac{N_0}{2T}\left[1+\frac{K-1}{2N}\right] \end{cases}$$

For the case when users are received at different power levels, assume $A = \sum_{i=1}^{K} P_i$ then Equation 5.21 can be simplified as follows,

$$var(I_{total}^{(1)}) = \begin{cases} (1-\zeta)^2(A-P_i) + \zeta^2\left[\frac{1}{4N^2}((K-2)A+P_i)\right] \\ +\zeta^2\frac{K-3}{8N^3}[(K-2)A+P_i] + \zeta^2\frac{1}{4N^2}P_i(K-2)(K-1) + \zeta^2\frac{3(K-1)P_i}{8}\left[\frac{3}{N^2}-\frac{2}{N^3}\right] \\ -\frac{2\zeta(1-\zeta)}{4N^2}[(K-2)A+P_i] + \frac{N_0}{2T}\left[1+\zeta^2\frac{K-1}{2N}\right] \end{cases}$$

Differentiating Equation 5.21 with respect to ζ results in,

$$\zeta_{min,i} = \frac{1+\frac{K-2}{2N}+\frac{(K-1)P_i}{2N(A-P_i)}}{\left[\begin{array}{l} 1+\frac{K-2}{2N}\left(2+\frac{K-3}{2N}\right)+\frac{(K-1)P_i}{A-P_i}\left(\frac{2K+9}{4N}+\frac{K-9}{4N^2}\right) \\ +\frac{K-1}{N}\frac{P_i}{A-P_i}\frac{1}{2\frac{E_b}{N_0}} \end{array}\right]} \tag{5.21}$$

For $P_1 = P_2 \ldots = P_K$, Equation 5.21 is,

$$\zeta_{min} = \frac{1+\frac{1}{2}\left(\rho-\frac{1}{N}\right)}{\left[\begin{array}{l} 1+\frac{1}{2}\left(\rho-\frac{2}{N}\right)\left(2+\frac{\rho-\frac{3}{N}}{2}\right) \\ +\frac{1}{4N}\left(2N\left(\rho+\frac{5}{N}\right)+\rho-\frac{9}{N}-1\right)+\frac{1}{2\frac{E_b}{N_0}} \end{array}\right]}$$

where, ρ defined as the load in the system, and is the ratio of number of users to processing gain. From Equation 5.21, it is seen that ζ_{min}, is dependent on the number of users in the system, the processing gain and the received signal powers of the users. Since the interference cancellation algorithm will be implemented at the base station; it has knowledge of the number of users in the system and the processing gain and the unknown quantity is the received signal power. In the later part of this section, a method to estimate the received signal powers of all users is presented. It will be shown that even in near-far systems the estimation method performs remarkably well. This suggests that the partial parallel coefficients ζ_{min} which minimize the total interference variance can be calculated at the base station. Hence, the method is feasible for practical implementation. For an asynchronous system, the values of ζ_{min} differ because of the fact that the cross-correlation between two PN sequences which are misaligned is $\frac{2}{3N}$ instead of $\frac{1}{N}$ for a synchronous system. Apart from this difference, there is no change in the implementation for a synchronous or an asynchronous system. The procedure to calculate values of ζ_{min} is discussed later in this section.

For the equal power case, Equation 5.22 suggests that for large values of $\frac{E_b}{N_0}$ the value of ζ_{min} is nearly independent of $\frac{E_b}{N_0}$. This is due to the fact that the reliability of the correlator outputs in the interference limited case is only dependent on the number of users and is independent of $\frac{E_b}{N_0}$ of the received signals. This is reflected in the values of ζ being independent of $\frac{E_b}{N_0}$ and is dependent only on the number of users in the system. Fig. 5.1 shows the value of ζ as a function of $\frac{E_b}{N_0}$ and number of users. Fig.

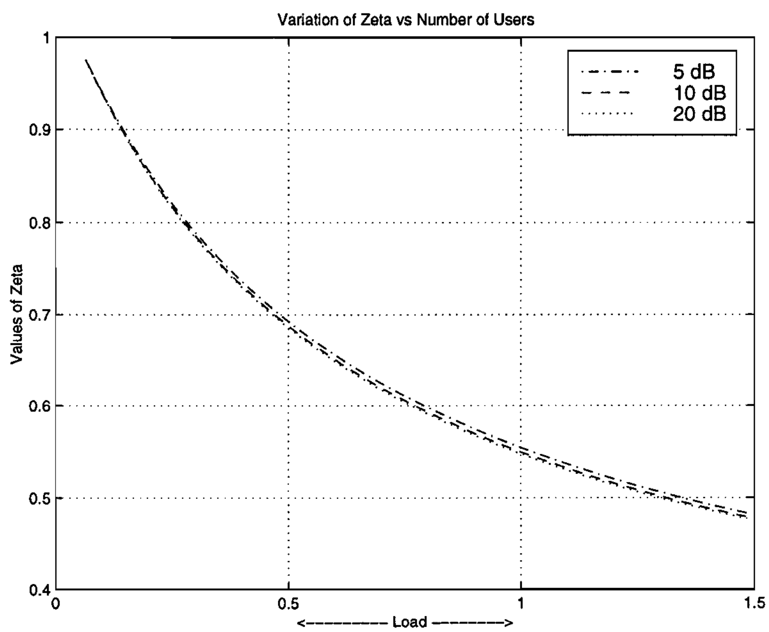


Figure 5.1: Variation of Zeta vs Number of Users

5.1 shows that the values of $zeta_{min}$ are close to one at low loads and decrease as the number of users increase. This can be explained intuitively; the total variance is a combination of interference estimates $I^{(1)}$ at the output of the first stage and unwanted interference $I_{uw}^{(1)}$ at the second stage. From the calculated variances $I^{(1)}$ and $I_{uw}^{(1)}$, it is seen that the variance of $I^{(1)}$ is a linear function of the load in the system. The variance of $I_{uw}^{(1)}$ is a non-linear function of the load and is proportional to the square of load. As the weights ζ determine the variance of interference that is passed to the subsequent stage, the weights ζ will be a function of the variances $I^{(1)}$ and $I_{uw}^{(1)}$. When the load is very less the weights ζ are high as the variance of $I_{uw,i}$ is less and hence interference $I^{(1)}$ should have a less weight. For higher loads, the value of ζ decreases since the variance of $I_{uw}^{(1)}$ increases with load and is greater than I_i for very high loads. The non-linear dependence of $I_{uw,i}$ on the load of the system gets reflected in the values of ζ_{min}. The simulated variance of $I^{(1)}$ and $I_{uw}^{(1)}$ are shown in Fig. 5.2. As predicted, it is noticed that the variance of $I_{uw}^{(1)}$ increases beyond the variance of $I^{(1)}$ at higher loads.

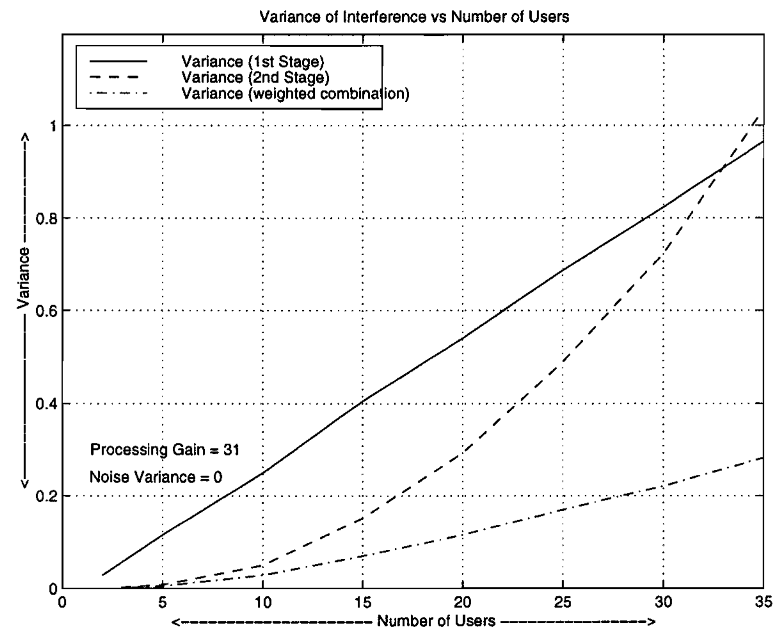


Figure 5.2: Interference Variance vs Number of Users

The calculation of ζ's in order to minimize the total interference variance after the first stage of cancellation is the first modification from the improved PIC algorithm.

In fact, the modified PIC algorithm differs in approach and implementation from improved PIC. In the improved PIC algorithm, where each stage estimates are a weighted combination of previous stage correlator estimates and current stage correlator estimates. Unlike the improved PIC, the modified PIC algorithm does not always combine the previous stage soft correlator estimates with the current correlator output. To present the modified algorithm, we define a stage called modified stage. Two stages or correlator banks are combined and called as a modified stage. For example, 4 stages of a multi-stage PIC will result in two modified stages. Thus, stage 1 and stage 2 are combined and called modified stage 1 and similarly stage 3 and stage 4 are combined and called modified stage 2. In the improved PIC, as stated earlier, each stage estimates are a weighted combination of previous stage correlator estimates and current stage correlator estimates. In the modified PIC algorithm, estimates within a modified stage are only combined. As an example, for a modified PIC the estimates of the third correlator stage $Z^{(3)}$ are not combined with the final decision estimates of the modified stage 1 Z_{f1}. In the improved PIC algorithm, the third stage correlator estimates are combined with the second stage estimates and so on. The final decision estimates Z_{f1} of modified stage 1 are used to estimate the received signal power. Instead of using the soft decision estimates Z_{f1} to calculate the interference, hard decisions are passed to the subsequent stage. Hence, the re-spread estimates after the first modified stage are,

$$\widehat{s}_i(t) = \sqrt{2\widehat{P}_i}\,\mathrm{sgn}(Z_{f1,i}(t))c_i(t)\cos(\phi_i(t)) \tag{5.22}$$

where $\widehat{P}_i$ is the estimated received signal power of user i and sgn(x) is the signum function defined as,

$$\mathrm{sgn}(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

The procedure for calculation of received signal powers is presented later in this section. The re-spread estimates obtained from Equation 5.22 are used to form new signal estimates which are passed to the first correlator bank of the next modified stage. The new signal estimates are,

$$\tilde{s}_i(t) = r(t) - \sum_{k=1,k\neq i} \sqrt{2\widehat{P}_i}\,sgn(Z_{f1,i}(t))c_i(t)\cos(\phi_i(t)) \tag{5.23}$$

These new signal estimates are correlated to obtain,

$$\begin{aligned} Z_i^{(3)} &= \sqrt{\frac{P_i}{2}}b_i + \sum_{k=1,k\neq i}^{K} \sqrt{\frac{P_k}{2}}b_k\rho_{ki} - \sum_{k=1,k\neq i}^{K} \sqrt{\frac{\widehat{P}_k}{2}}sign(Z_{f1,k})\rho_{ki} + \chi_i \\ &= D_i + I_i^{(1)} - \widehat{I}_i + \chi_i \end{aligned} \tag{5.24}$$

The evaluation of $\widehat{I}_i$ is difficult and hence to calculate parallel weighing coefficients which minimize the interference variance is cumbersome. The fact that the bit decisions of all users at this stage are not independent makes it difficult to calculate the variance of estimated interference. Hence, the performance of the receiver is analyzed through simulations. The complex base-band simulation model is explained in section 5.2. As in the case of modified stage 1, where the estimates $Z^{(1)}$ are used to estimate $I_i^{(1)}$, the estimates $Z_i^{(3)}$ are used to estimate interference $I_i^{(1)}$. Similar to the first modified stage, where the estimates $\widehat{I}_i^{(1)}$ are subtracted from $Z^{(1)}$ to form modified estimates $Z^{(1)}_{mod,i}$ the calculated MAI estimates from $Z_i^{(3)}$ are subtracted from $Z^{(1)}$ to form a new estimate.

$$\begin{aligned} Z_i^{(4)} &= Z_i^{(1)} - \tilde{I}_i \\ &= Z_i^{(1)} - \frac{2}{T}\int_0^T \sum_{k=1,k\neq i} Z_k^{(3)} c_k(t)\cos(\phi_k)c_i(t)\cos(\phi_i)dt \end{aligned} \tag{5.25}$$

Both $Z_i^{(3)}$ and $Z_i^{(4)}$ are combined to yield,

$$Z_{f2,i} = \zeta Z_i^{(3)} + (1-\zeta)Z_i^{(4)} \tag{5.26}$$

where, $Z_{f2,i}$ is the final decision estimate of modified stage 2. The parallel cancellation factors used are ζ_{min}, which are calculated in the first modified stage. As a matter of fact, these partial cancellation factors ζ_{min}'s might not minimize the total interference variance at the output of the second modified stage. As the interference estimate $\tilde{I}_i$ has residual interference which is non-linear in the users, the values of ζ are valid as a first approximation.

For any modified stage the algorithm is as follows :

- For each modified stage the inputs are the correlator outputs of first stage $Z^{(1)}$ and the re-spread estimates of previous stage.
- The re-spread estimates are passed to their respective correlators to obtain correlator estimates $Z^{(m)}_{corr}$.
- The correlator estimates $Z^{(m)}_{corr}$ are re-spread and are used to estimate interference to the first correlator estimates $Z^{(1)}$. The calculated interference estimates are subtracted from $Z^{(1)}$ to form a modified estimate $Z^{(m)}_{mod}$.
- Both the estimates are combined to obtain a final estimate.

$$Z_f^{(m)} = \zeta Z^{(m)}_{mod} + (1 - \zeta) Z^{(m)}_{corr} \tag{5.27}$$

  where the values of ζ are those calculated for the first modified stage.

- The final correlator estimates $Z_f^{(m)}$ are used to calculate the received signal powers and these estimates are used to calculate the new signal that is to be passed to the subsequent stage.

Fig. 5.3, shows a block diagram of a modified PIC receiver. An extended block diagram of the modified stage is shown in Fig. 5.4.

A modified PIC (MPIC) algorithm was presented in this section. The MPIC algorithm is based on the concept of partial parallel interference cancellation. In the proposed algorithm, the partial cancellation coefficients are chosen to minimize the variance after the first stage of cancellation or two correlator stages. It is shown that for equal power case, the partial correlation coefficients are nearly independent of $\frac{E_b}{N_0}$ and depend only on the number of users, processing gain. In the next section, the performance of MPIC receiver is analyzed through simulations and is compared to already proposed algorithms.

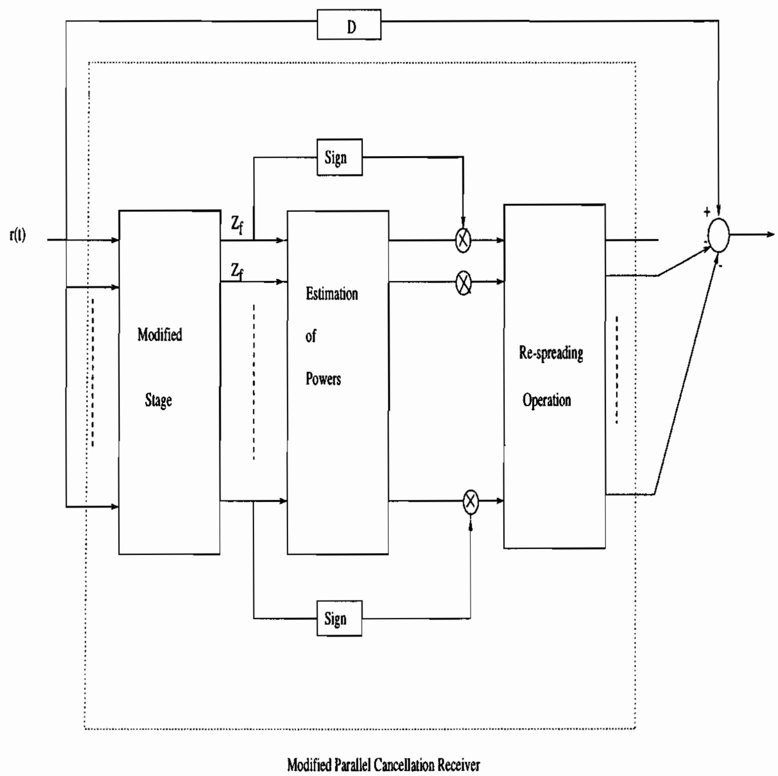
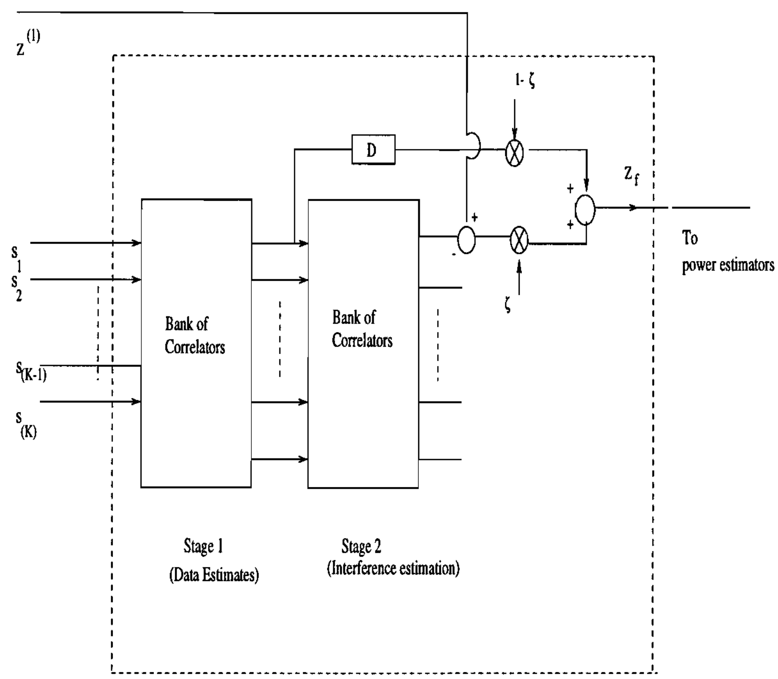


Figure 5.3: Block Diagram of Modified PIC



Block Diagram of the Modifed Stage

Figure 5.4: Block Diagram of modified stage

5.1.2 Estimation of received signal powers

As stated earlier, we assume that the receiver does not have a knowledge of received signal powers. For a MPIC receiver, it turns out that the partial parallel cancellation coefficients are dependent on the received signal powers. In a multi-user system as in the CDMA system, it is a little difficult to calculate the received signal powers of all users. As the partial cancellation coefficients are dependent on the received signal powers for an MPIC algorithm, it is necessary to estimate the received signal power as accurately as possible.

A windowed approach is developed to estimate the user's received signal powers. It is assumed that the received signal power is constant for a period of M symbols. This is simply the assumption of a slow fading channel (for the simulations a window length of 100 symbols is assumed). The received signal powers are estimated by calculating the variance of these M symbols,

$$\begin{aligned}
\widehat{P}_i &= E\left[\left(Z^2\right)^{(1)}_{f1,i}\right] \\
&= \frac{1}{M}\sum_{j=1}^{M}\left(Z^2\right)^{(1)}_{f1,i} \qquad (5.28)
\end{aligned}$$

As the estimates $Z_{f1,i}$'s are corrupted by MAI, for high system loads, the calculated estimates will be erroneous. A method to refine these estimates is proposed in this section. Expanding Equation 5.28 and substituting for $Z_{f1,l}$,

$$\begin{aligned}
\widehat{P}_i &= E\left[\left(Z^2\right)^{(1)}_{f,i}\right] \\
&= E\left[(D_i + I_{total,i})^2\right] \\
&= E[D_i^2] + E[I_{total,i}^2] + 2E[D_i I_{total,i}] \qquad (5.29) \\
&= P_i + var(I_{total,i}) - 2\zeta\frac{(K-1)}{2N}P_i \qquad (5.30)
\end{aligned}$$

Let $var(I_{total,i}) = cP_i$ and $2\zeta\frac{(K-1)}{2N}P_i = dP_i$; substituting in Equation 5.30 results in,

$$\widehat{P}_i = P_i(1 + c - d) \qquad (5.31)$$

The raw power estimates are $\widehat{P}_i$. These raw power estimates are refined as follows,

$$\begin{aligned}
\tilde{P}_i &= \widehat{P}_i - c\widehat{P}_i + d\widehat{P}_i \qquad (5.32) \\
&= P_i(1 + c - d) - c(1 + c - d)P_i + d(1 + c - d)P_i \\
&= P_i\left(1 - (c - d)^2\right)
\end{aligned}$$

If c and d are less than one then the refined power estimates $\tilde{P}$ are closer to the actual values. For the synchronous system, this results in

$$\tilde{P}_i = \left\{ \begin{array}{l}
\widehat{P}_i + \zeta\frac{K-1}{N}\widehat{P}_i + \zeta(1-\zeta)\frac{1}{2N^2}\left((K-2)\sum_{j=1}^{K}\widehat{P}_j + \widehat{P}_i\right) \\
-(1-\zeta)^2\frac{1}{2N}\left(\sum_{j=1}^{K}\widehat{P}_j - \widehat{P}_i\right) - \zeta^2\frac{1}{4N^2}\left((K-2)\sum_{j=1}^{K}\widehat{P}_j + \widehat{P}_i\right) \\
-\zeta^2\frac{1}{8N^3}(K-2)(K-3)\sum_{j=1}^{K}\widehat{P}_j + \zeta^2(K-3)\frac{\widehat{P}_i}{8N^3} - \zeta^2(K-2)(K-1)\frac{\widehat{P}_i}{4N^2} \\
-\zeta^2\frac{3(K-1)}{8}\left(\frac{3}{N^2} - \frac{2}{N^3}\right)\widehat{P}_i
\end{array}\right.$$

The performance of the refined power estimation method is shown in Fig. 5.5. The

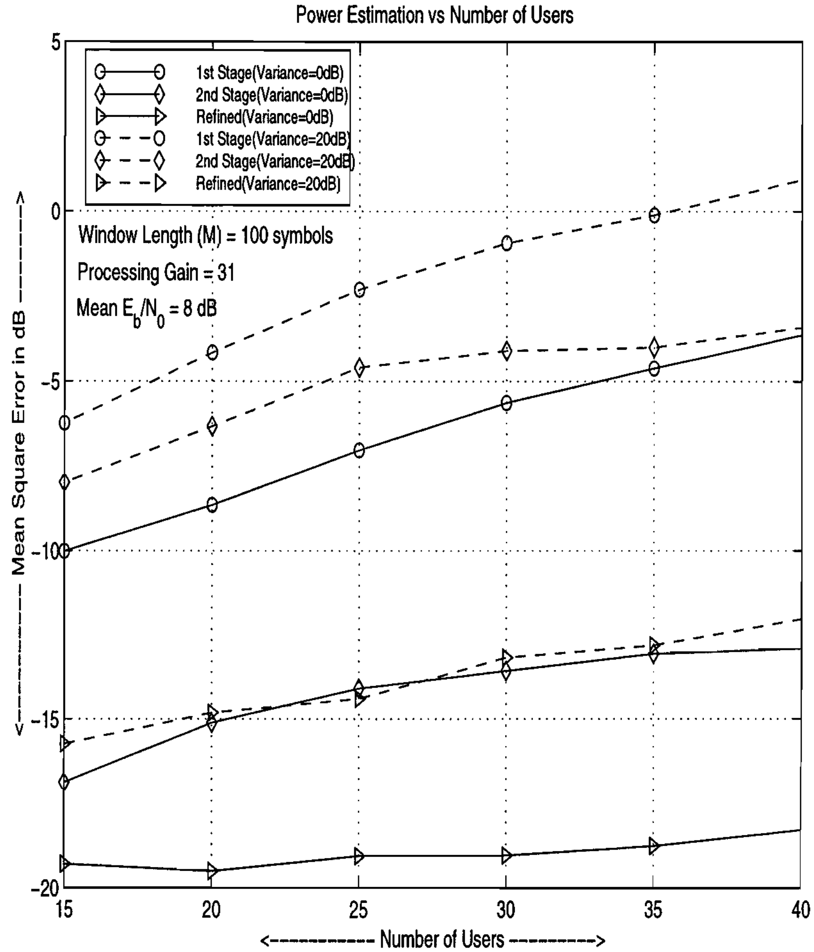


Figure 5.5: Comparison of power estimates

mean square error of the estimates as a function of number of users is plotted in Fig. 5.5. Two cases are considered to test the performance of the estimation method. The first case that is considered is the equal received signal power case. For this case, the mean square error (MSE) of the refined power estimates is close to -20 dB (the window length was 100). For the same case, the MSE of the estimates calculated from the correlator outputs of stage 1 increases linearly with number of users. For the equal power case, the MSE of the estimates calculated from the final decision estimates of modified stage 1 $Z_{f1,i}^{(1)}$ also increases with number of users in the system. The second case considered was a near-far environment system. The users received signal powers have a lognormal distribution with a variance of 20 dB. For this case too, the MSE of the refined power estimates is around -10 dB at 40 users for a processing gain of 31. Similarly, MSE when 1st stage correlator outputs are considered increases with number of users and is around 1 dB at 40 users for a processing gain of 31. The results show that a significantly better received signal power estimation is performed by the new estimation method.

In fact, these received signal power estimates are used in calculation of ζ_{min}. Initially, ζ_{min} values are calculated by using the received signal power estimates that are calculated from the 1st stage correlator outputs. These values are used to calculate the refined power estimates. Subsequently, these refined power estimates are used to calculate values of ζ_{min}. This procedure has an inherent delay of M symbols assuming M is the window length. The method for estimating the received signal powers is outlined in this section.

5.2 Complex base-band simulation model

Before the results are presented and complexity issues of the receiver are addressed, the simulation model developed for simulating the system as well as the receiver is presented in this section.

A complex simulation model was developed for the performance analysis of the system. All the simulations were carried out in baseband, complex envelope form [54]. This section describes the model for transmitter, channel and the receiver used for simulation in their low pass equivalent forms.

The transmitted signal of each user described in 5.1, can be expressed in its complex envelope form as,

$$\begin{aligned} s_i(t) &= Re\{\sqrt{2P_i}b_i(t)c_i(t)e^{j(\omega_c t+\phi_k)}\} \\ &= Re\{\tilde{s}_i(t)e^{j\omega_c t}\} \end{aligned} \tag{5.33}$$

where, $\tilde{s}_i(t)$ is the complex envelope of the transmitted signal $s_i(t)$. For simulation of the complex baseband transmitted signal, it is separated into its in-phase and quadrature component as,

$$\begin{aligned}
\tilde{s}_{i,I} &= \sqrt{2P_i}b_i(t)c_i(t)\cos(\phi_i) \\
\tilde{s}_{i,Q} &= \sqrt{2P_i}b_i(t)c_i(t)\sin(\phi_i) \qquad (5.34)
\end{aligned}$$

For all the simulations, PN codes were used with a unique code for each user. The processing gain of the system can be varied and similarly the number of samples per chip. Long PN codes or short PN codes can be used for a given simulation, depending on the simulation run.

A simple AWGN channel is considered for the analysis purposes. A complex Gaussian sequence with a specified variance is generated which is then added to the sum of generated complex low-pass equivalent signals.

The receiver implementation is also similar to the transmitter implementation. For calculating BER, the total number of symbols that are transmitted through the system is at least ten times the desired BER. For example, for a required BER of 10^{-3}, the total number of symbols that are transmitted through the system is at least 100,000 symbols. For all the simulations, a pure AWGN channel is considered and also assume a synchronous system.

5.3 Modified PIC - Results and Comparisons

In this section, the performance of a MPIC receiver is presented. The MPIC receiver is simulated using the complex baseband simulation model presented in section 5.2. It is assumed that the receiver has perfect knowledge of the phases and delays of all the users apart from the knowledge of user PN sequences. MPIC receiver's performance is compared with already proposed receivers. All the simulations were performed for a synchronous system. The algorithm is same for a synchronous system and an asynchronous system. The values of ζ_{min} will be different for a synchronous and asynchronous system because PN codes cross-correlations are different. Apart from this change, no other modification is required if the receiver is operating in an asynchronous system.

The BER vs number of users for for a synchronous system at an $\frac{E_b}{N_0} = 8$dB is shown in Fig. 5.6. As shown in Fig. 5.6, for a required BER of 10^{-3} a conventional receiver

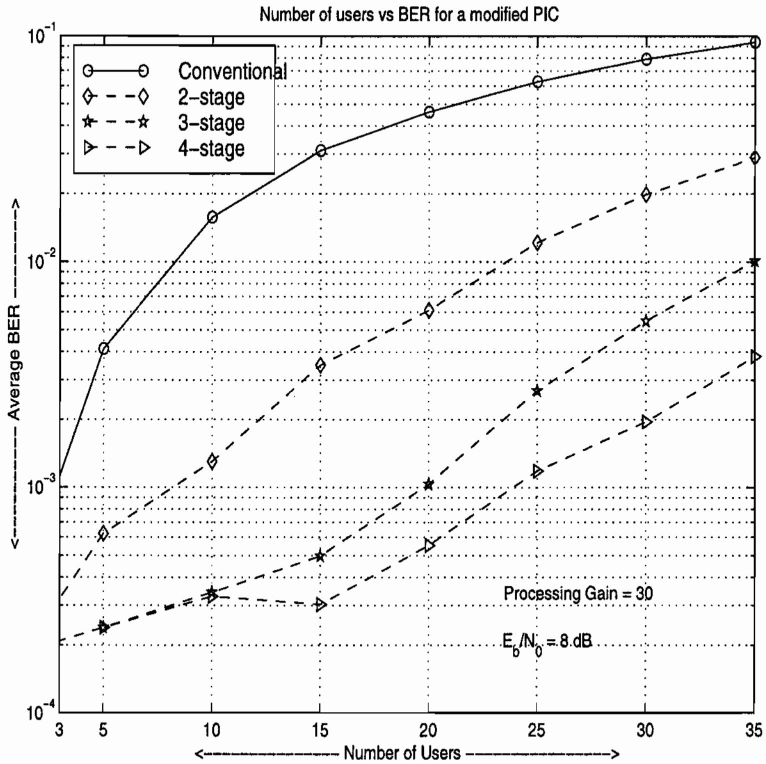


Figure 5.6: Number of users vs BER for modified PIC receiver

supports 3 users. For the same required BER a 4-stage modified receiver supports around 25 users, a gain of around 8 relative to a conventional receiver. Fig. 5.7, compares the performance of a 2-stage partial PIC receiver where the partial cancellation coefficients are optimized, as proposed by Renucci [52] with the MPIC receiver. It is assumed that the partial PIC has knowledge of user's power and noise power and the modified PIC has no knowledge of the user's powers. The comparison indicates that both the receivers have similar performance at light loads. The performance of MPIC is slightly better than Renucci's [52] receiver. Fig. 5.8, compares the performance of a 4 stage partial PIC with a modified PIC. For the partial PIC, instead of using soft correlator outputs for re-spreading, the estimated received signal powers are used along with hard decisions so that similar operations are performed by both the receivers which are being compared. From Fig. 5.8, it is seen that a 4-stage modified PIC has better per-

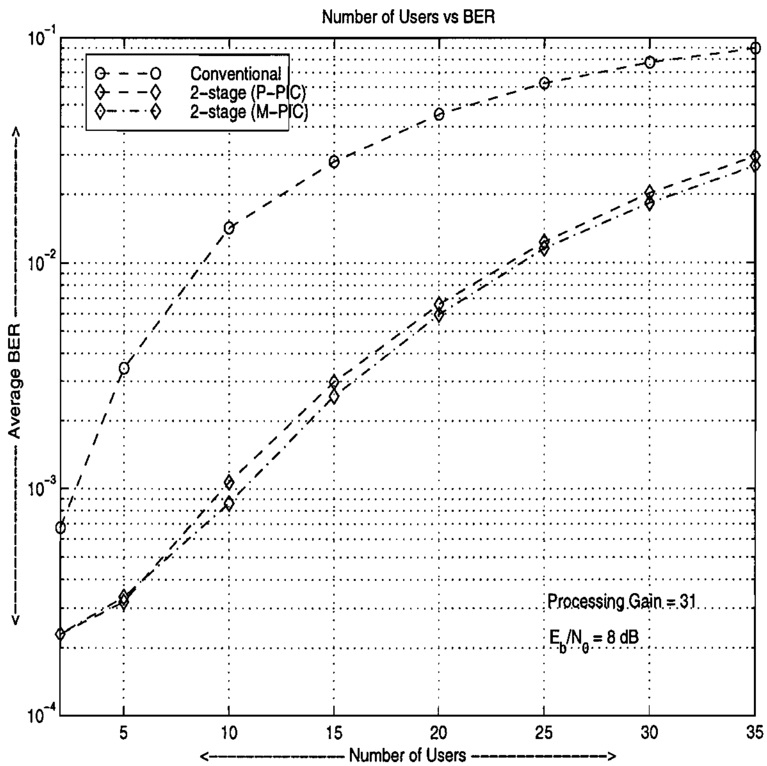


Figure 5.7: Comparison of Modified PIC with optimized Partial PIC

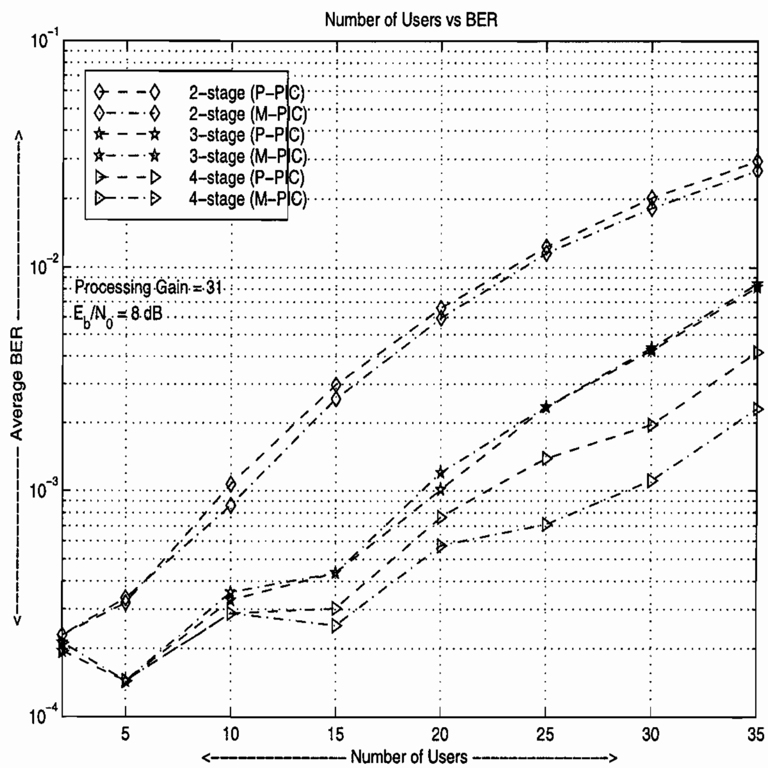


Figure 5.8: Comparison of Modified PIC with optimized Partial PIC

formance than a 4-stage partial PIC. Fig. 5.9 compares the performance of a multistage PIC, partial PIC (P-PIC) and modified PIC for 3 -stages. Fig.5.9 suggests that a three

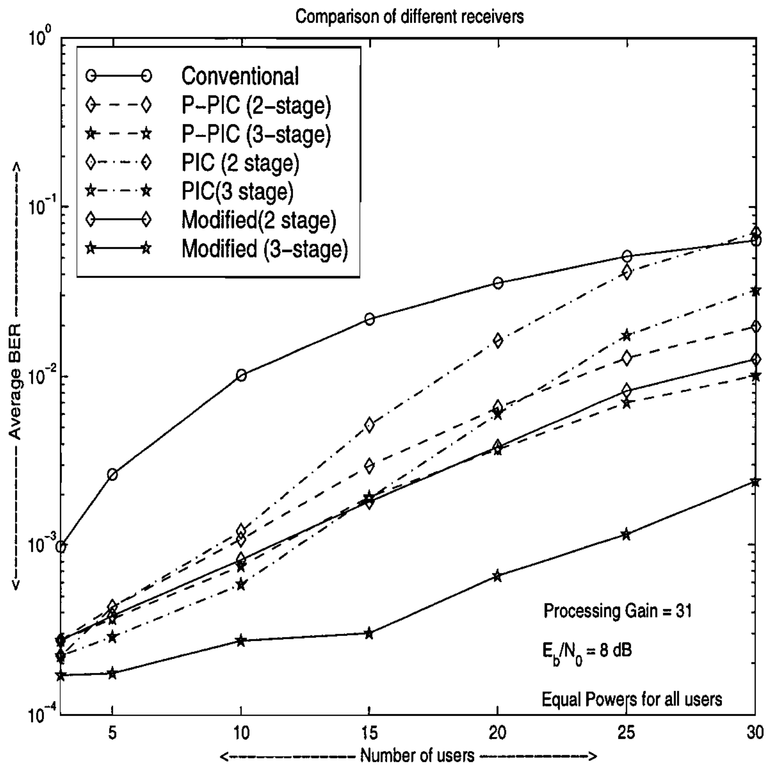


Figure 5.9: Comparison of different receivers

stage MPIC has better performance than a partial PIC or a multi-stage PIC receivers.

The performance of M-PIC for different processing gains is considered in order to obtain an idea of the MPIC receiver's performance as processing gain varies. The MPIC receiver's performance is simulated for four different processing gains. The four processing gains that are considered are 15, 30, 45 and 60 for an $\frac{E_b}{N_0}$ = 8 dB and in addition to these values a processing gain of 94 was considered for $\frac{E_b}{N_0}$ = 10 dB. The performance of the receiver for each processing gain is shown in Figs. 5.10, 5.11, 5.12 and 5.13.

For a voice service the generally considered QoS is a BER of 10^{-3}. The number of users that each system supports for a BER of 10^{-3} are compared. The number of users that can be supported is shown if Fig. 5.14

A similar comparison is performed for the case when interference variance is equal

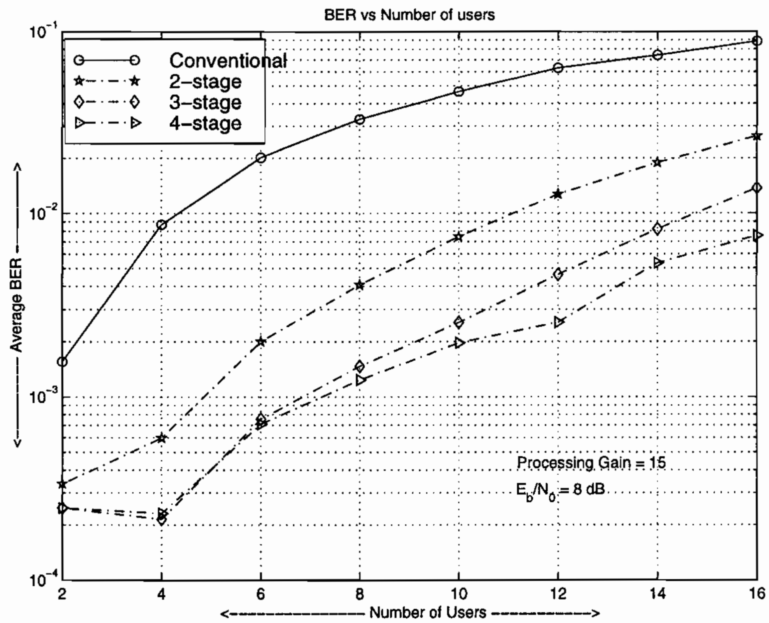


Figure 5.10: Number of users vs BER for processing gain = 15

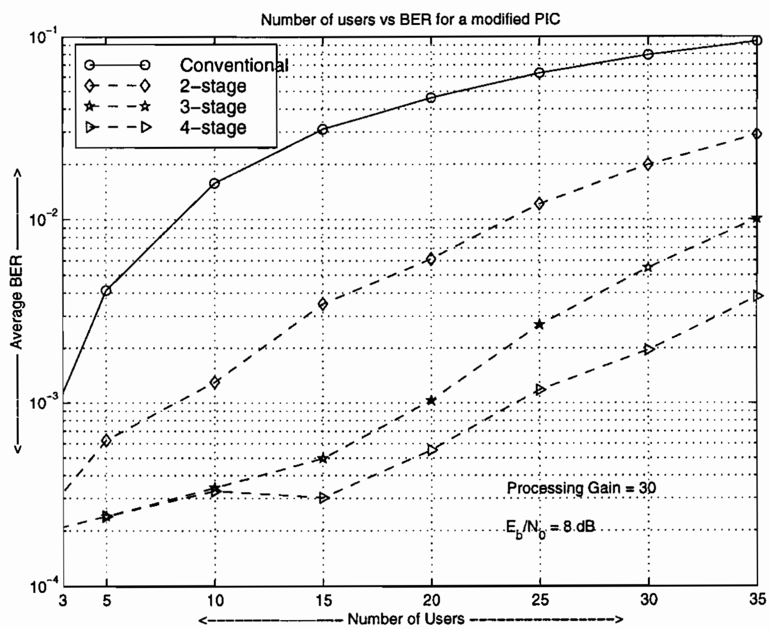


Figure 5.11: Number of users vs BER for processing gain =30

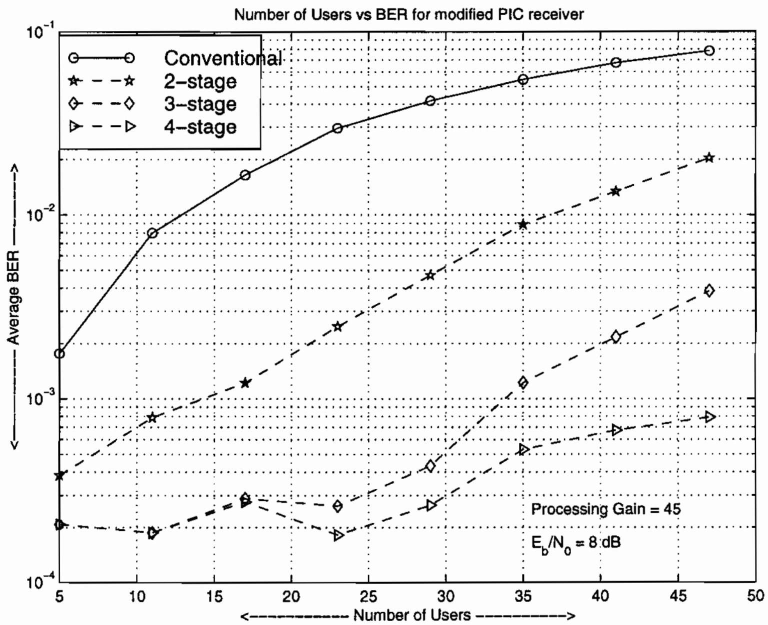


Figure 5.12: Number of users vs BER for processing gain = 45

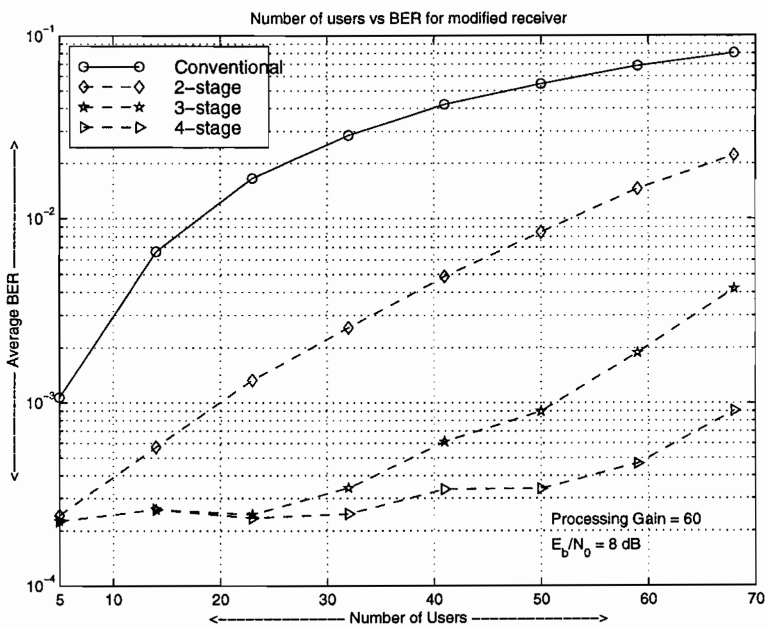


Figure 5.13: Number of users vs BER for processing gain = 60

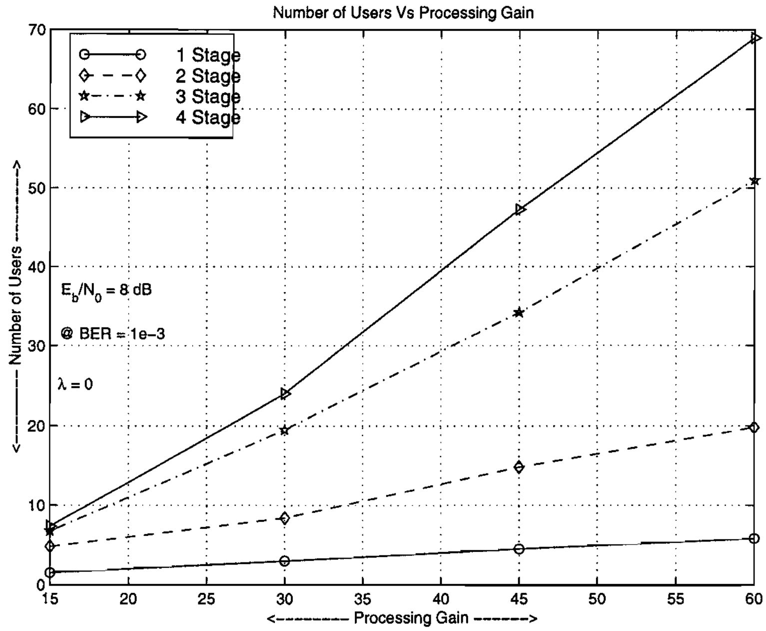


Figure 5.14: Number of users supported for BER of 10^{-3}

to noise variance [37] at the desired QoS. This implies that for a conventional receiver, at a BER of 10^{-3} the interference variance is equal to noise variance. The number of users as a function of processing gain is shown in Fig. 5.15. and the gain obtained at each stage is shown in Fig. 5.16.

The above results, suggest that a four stage MPIC receiver has a considerable capacity gain relative to a conventional receiver. The capacity gain of a MPIC receiver relative to a conventional receiver depends on the processing gain apart from $\frac{E_b}{N_0}$. The MPIC's performance is compared to the receiver proposed by Divsalar *et al.* The number of users supported by a 3-stage improved PIC receiver for a degradation of 1 dB is around 81 users. For the same parameters and a required BER of 10^{-2}, the number of users supported by a 3-stage MPIC receiver is around 99 users; a gain of 18 users relative to improved PIC algorithm at a BER of 10^{-2}. The performance of the MPIC receiver for the same parameters is shown in Fig. 5.17 Thus the MPIC receiver achieves better performance of an improved parallel cancellation receiver without the complexity associated with the improved PIC algorithm. The MPIC receiver is feasible to be implemented practically and does not need the knowledge of received signal powers

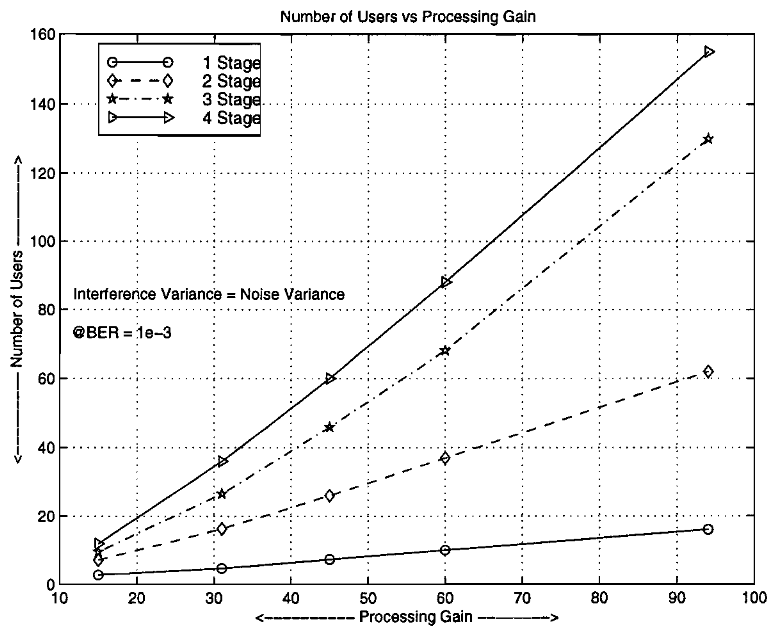


Figure 5.15: Number of users for different receivers for BER of 10^{-3}

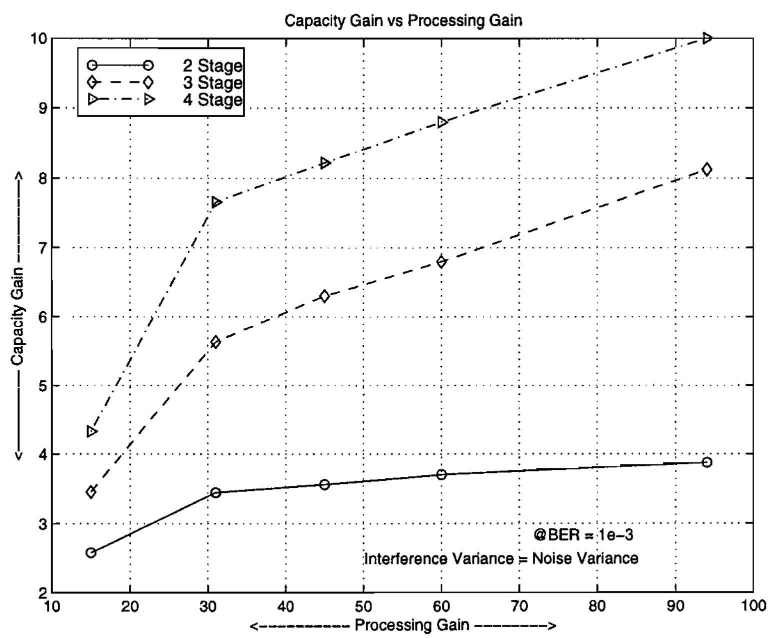


Figure 5.16: Capacity gains for different receivers at 10^{-3}

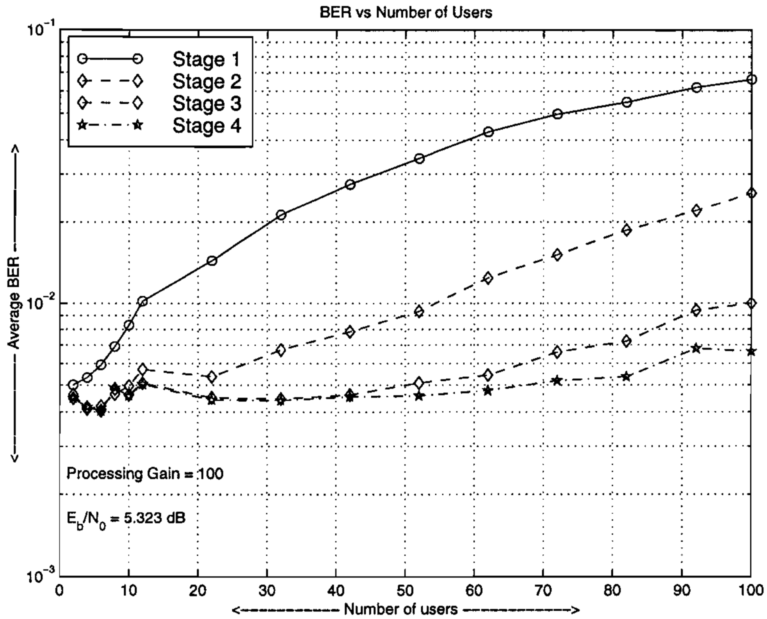


Figure 5.17: Performance of MPIC at an $\frac{E_b}{N_0}$=5.323dB

of users.

The performance of the MPIC receiver for the multiple-cell system model proposed in chapter 2, is considered. The simulation is performed for various values of λ which as already defined is the ratio of out-of-cell interference to in-cell interference. The BER curves for various values of λ is shown in Fig. 5.18.

The comparison with the optimal receiver at a BER of $1\text{x}10^{-3}$ and a processing gain of 31 and an $\frac{E_b}{N_0} = 10$ dB is shown in the table,

λ	Conventional	4-stage M-PIC	Gain of M-PIC	Optimal Rcver Gain
0	4.9	36	7.35	∞
0.1	4.1	22.5	5.48	11.00
0.3	3.7	11.5	3.10	4.33
0.5	3	8.5	2.83	3.00

In this section, the performance of a modified parallel interference cancellation receiver is presented. The results indicate that a 4-stage MPIC receiver supports considerably large number of users than can be supported by a conventional receiver for a given BER. It is noticed that the MPIC receiver provides a capacity gain which is dependent on the processing gain of the system.

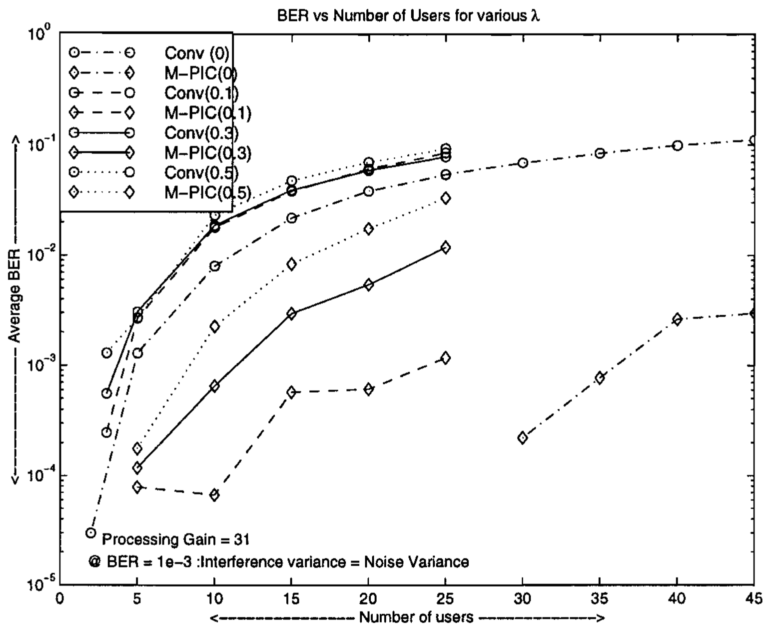


Figure 5.18: Performance curves for various values of λ

5.4 Complexity Of MPIC Receiver

The complexity of the modified parallel interference cancellation receiver and some implementation issues are discussed in this section. The complexity of the receiver is measured in terms of the number of operations per bit. For the modified receiver, the complexity can be calculated assuming that the receiver has phase and timing information. We assume that the received signal is available in base-band form at the input of the first stage. The operations that are performed in a single modified stage are; correlation of the received base-band signal to obtain $Z^{(m)}$, re-spreading of the correlator estimates to obtain $\widehat{s}$, estimating the interference to the correlator estimates to obtain $\widehat{I}$, modifying the correlator estimates to obtain Z_{mod} followed by combining the correlator and modified estimates to obtain the final estimates Z_{fin}. The final estimates are re-spread again and these re-spread estimates are subtracted from the received base-band signal to obtain estimates $\widehat{s}^{s+1}$ which are passed to the subsequent modified stage. Power estimates are calculated once every M symbols (assuming a window length of M). Assuming that most of the operations are complex, the complexity of the receiver per bit per user is; correlation operation is O(2NNs), the complexity of spreading operation is O(2NNs) and the complexity of the estimation process is O(2NNs) , complexity of interference estimation is O(2NNs), the complexity of combining operation is O(3) and re-spreading operation is O(3NNs). Since, power estimation is performed once every M symbols the complexity of power estimation is O(4) and the complexity of signal estimation to be passed to the next stage is, O(2NNs). Thus the complexity per bit per user for a single stage is, O(11NNs). For K users and 2 stages of the receiver, the complexity is O(22KNNs) and hence the complexity of the receiver is linear in the number of users. The complexity of MPIC when compared to a conventional receiver is of the order of 10, which is moderate when compared to an optimal receiver whose complexity is of order of $O(2^K)$. Thus an MPIC receiver provides a significant capacity gain is obtained for a moderate increase in the complexity of the system.

5.5 Chapter Summary

In this chapter, a modified parallel interference cancellation receiver is presented. Each modified stage of the receiver contains two correlator banks, the first correlator bank estimates the data decisions and the second correlator bank estimates the MAI to the correlator estimates of the first stage. These two estimates are weighted and combined. The weighing coefficients are dependent on the number of users and received signal powers of the users. A method for estimation of the received signal powers of the users is also presented in this chapter. The user capacity gain of the MPIC receiver relative to a conventional receiver is dependent on the processing gain and $\frac{E_b}{N_0}$. The order of capacity gain of the MPIC receiver for a single-cell varies from 4-10 for processing gains of 15-94. The complexity of the receiver is shown to be linear in the number of users. The modified PIC receiver is practical to implement under the assumption that the receiver has knowledge of phase and delay information.

Chapter 6

Conclusions And Future Work

6.1 Summary Of Results

This technical report developed a MPIC receiver based on the concept of partial interference cancellation. The capacity gains relative to a conventional receiver in the order of 4-10 have been obtained for a 4-stage interference cancellation receiver. The partial cancellation coefficients are calculated in order to minimize the total interference variance after the first two stages. The idea of the MPIC receiver stems from the partial parallel interference cancellation proposed by Divsalar *et al.* [1]. The algorithm implementation approach presented by Divsalar *et al.* is complex and involved non-linear decision devices. Compared to the improved parallel cancellation approach, the MPIC receiver's approach to interference cancellation is simple and straight forward. In the improved partial cancellation approach, it is assumed that the received signal powers of the users are known. The MPIC receiver does not assume the knowledge of the received signal powers. On the contrary, a method for estimating the received signal powers is developed. It is shown that the received signal power estimation method's performance is reasonably good even in case of a near-far environment.

The motivation of this technical report was to develop an interference cancellation algorithm based on either a SIC or PIC scheme. The idea is to develop an IC receiver based on a scheme which yields larger user capacity. Since, there was no generic comparison of the PIC and SIC schemes, we developed an information theoretic approach to evaluate the user capacity of CDMA system. Although the approach could be applied to any receiver scheme, it was not feasible to be implemented for a PIC scheme. As the approach could not be applied directly to a PIC scheme, the SIC scheme's performance was compared to an ideal IC. It was seen that a SIC scheme has a performance which is considerably worse than a PIC scheme. This prompted us to choose a PIC scheme as the receiver structure for modification. This analysis was carried out in chapter 3.

In chapter 4, already proposed PIC schemes performance was reduced. The improved parallel cancellation approach proposed by Divsalar *et al.* was shown to provide a gain of around 9 at a BER of 10^{-2} for a 1 dB degradation compared to a single channel case. Although, the idea of the algorithm was simple, the implementation approach proposed by them was complex and the partial cancellation coefficients are calculated using a simulation approach.

The MPIC receiver was presented in chapter 5. Unlike the improved PIC, the MPIC algorithm is straightforward to implement and is less complex. The partial cancellation coefficients are calculated in order to minimize the variance of total interference after a single stage of cancellation. A method to calculate the received signal powers of users is presented and is shown to have considerably better performance when simply the correlator estimates are used for estimation of received signal powers. The calculated signal power estimates coupled with the hard decisions of the final decision estimates at the end of second stage are passed to the third stage. In the improved PIC algorithm, the data decision output at each stage is a weighted combination of the current estimates and the previous stage estimates. In case of the MPIC algorithm, the third stage estimates are not combined to the second stage estimates as is done in improved PIC. In fact, the third stage estimates are used to estimate interference and both these estimates are again combined to form a new estimate. For a single-cell, the number of users that a 4-stage MPIC can support for a required BER is around 9 times the user capacity of a conventional receiver. For a multi-cell system, employing a MPIC receiver at the base station results in the base station supporting around 2 times the capacity of a base station employing a conventional receiver for a required BER of 10^{-3}.

A complex baseband simulation model was developed to implement the receiver and extensive simulations have been performed to validate the results. The simulation model accurately models the DS-CDMA system and can be built upon to model extensive simulations in future.

In conclusion, a new modified parallel interference cancellation receiver has been proposed in this technical report. A method to estimate the received signal powers of the users as part of the cancellation procedure is also developed. This technical report also developed an information theoretic approach to calculate user capacity for a DS-CDMA system employing any receiver.

6.2 Future Work

The receiver's performance was considered only for AWGN channel which is not the channel that is normally encountered in a cellular system. Hence it would be of great interest to test the system for a Rayleigh channel or Rician channel. Throughout the simulations it is assumed that the receiver has a perfect knowledge of the timing and phase parameters of individual users. In a practical setting, the timing and phase estimates cannot be calculated accurately, this needs to be factored into the receiver and observe the performance of the system. An interesting aspect would be to investigate the application of error control coding to the transmitter and receiver and investigate the performance. The simulation model that we developed is a basic model and could be used to build additional models of interest on top of it for future work.

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Appendix A

Calculation of Variance of Total Interference

From chapter 5, the variance of $I_{total,i}$ is ,

$$var(I_{total,i}^{(1)}) = Var\left(I_i^{(1)}(1-\zeta) - \zeta I_{uw,i}^{(1)} - \zeta\chi_{p,i} + \chi_i\right) \tag{A.1}$$

$$= \left\{ \begin{array}{l} E\left[\left(I_i^{(1)}(1-\zeta) - \zeta I_{uw,i}^{(1)} - \zeta\chi_{p,i} + \chi_i\right)^2\right] - \\ \left[E\left(I_i^{(1)}(1-\zeta) - \zeta I_{uw,i}^{(1)} - \zeta\chi_{p,i} + \chi_i\right)\right]^2 \end{array} \right.$$

Expanding Equation A.2 results in,

$$var(I_{total}^{(1)}) = \left\{ \begin{array}{l} (1-\zeta)^2 E[(I_i^{(1)}).^2] + (\zeta)^2 E[(I_{uw,i}^{(1)})^2] + (\ zeta)^2 E[(\chi_{p,i})^2] + E[(\chi_i)^2] \\ -2(1-\zeta)\zeta E[I_i^{(1)} I_{uw,i}^{(1)}] - 2(1-\zeta)E[I_i^{(1)}\chi_{p,i}] + 2(1-\zeta)E[I_i^{(1)}\chi_i] \\ +2(\zeta)^2 E[I_{uw,i}^{(1)}\chi_{p,i}] - 2\zeta E[I_{uw,i}^{(1)}\chi_i] - 2\zeta E[\chi_i\chi_{p,i}] \\ -\left[E\left(I_i^{(1)}(1-\zeta) - \zeta I_{uw,i}^{(1)} - \zeta\chi_{p,i} + \chi_i\right)\right]^2 \end{array} \right.$$

The variance of each term is,

A.0.1 Variance of $I^{(1)}$

The variance of $I^{(1)}$ is calculated in chapter 5 and is,

$$var(I_i^{(1)}) = \frac{\sum_{k=1,k\neq i}^{K} P_k}{2N} \tag{A.2}$$

A.0.2 Variance of $I_{uw}^{(1)}$

The unwanted interference term $I_{uw}^{(1)}$ is,

$$\begin{aligned} I_{uw,i}^{(1)} &= 2\sum_{k=1,k\neq i}^{K}\int_{t=0}^{T}\sum_{j=1,j\neq k}^{K}\rho_{jk}\sqrt{\frac{P_j}{2}}b_j\cos(\phi_k-\phi_j)c_k(t)\cos(\omega_c t+\phi_k)c_i(t)\cos(\omega_c t+\phi_i)dt \\ &= \sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq k}^{K}\rho_{jk}\rho_{ki}\sqrt{\frac{P_j}{2}}b_j\cos(\phi_k-\phi_j)\cos(\phi_k-\phi_i) \end{aligned} \tag{A.3}$$

It turns out that the mean of $I_{uw}^{(1)}$ is zero. Hence variance of $I_{uw}^{(1)}$ is,

$$\begin{aligned} var(I_{uw}^{(1)}) &= E\left[\left(I_{uw}^{(1)}\right)^2\right] \\ &= E\left[\left(\sum_{k=1,k\neq i}^{K}\sum_{j=1,j\neq k}^{K}\rho_{jk}\rho_{ki}\sqrt{\frac{P_j}{2}}b_j\cos(\phi_k-\phi_j)\cos(\phi_k-\phi_i)\right)^2\right] \end{aligned} \tag{A.4}$$

The expectation operation cannot be directly taken inside the summation because the terms are not completely dependent. Hence, to calculate the variance of $I_{uw}^{(1)}$, we evaluate each terms independently. Since, the terms with index i overlap Equation A.4 is written as, (assume $i = 1$), Let C and D be defined as,

$$\begin{aligned} C &= \sum_{k=2}^{K}\sum_{j=2,j\neq k}^{K}\rho_{jk}\rho_{k1}\sqrt{\frac{P_j}{2}}b_j\cos(\phi_k-\phi_j)\cos(\phi_k-\phi_1) \\ D &= \sum_{k=2}^{K}\rho_{k1}\rho_{1k}\sqrt{P_1}b_1\cos(\phi_k-\phi_1)\cos(\phi_k-\phi_1) \end{aligned} \tag{A.5}$$

$$var(I_{uw}^{(1)}) = E\left[C^2\right] + E\left[D^2\right] + 2E\left[CD\right] \tag{A.6}$$

The variance of D is,

$$
\begin{aligned}
E\left[D^2\right] &= E\left[\left(\sum_{k=2}^{K} \rho_{k1}\rho_{1k}\sqrt{P_1}b_1\cos^2(\phi_k-\phi_1)\right)^2\right] \\
&= P_1\sum_{k=2}^{K} E\left[\rho_{jk}^4\cos^4(\phi_k-\phi_1)\right] \\
&= (K-1)P_1\frac{3}{8N^2}\left(\frac{3}{N^2}-\frac{2}{N^2}\right) \qquad \text{(A.7)}
\end{aligned}
$$

The variance of C is,

$$
\begin{aligned}
E\left[C^2\right] &= E\left[\left(\sum_{k=2}^{K}\sum_{j=2,j\neq k} \rho_{jk}\rho_{k1}\sqrt{\frac{P_j}{2}}b_j\cos(\phi_k-\phi_j)\cos(\phi_k-\phi_1)\right)^2\right] \\
&= \left\{\begin{array}{l} \sum_{k=2}^{K} E\left[\sum_{j=2,j\neq k}^{K} \rho_{jk}\rho_{k1}\sqrt{P_j}b_j\cos(\phi_k-\phi_j)\cos(\phi_k-\phi_1)2\right] \\ +\sum_{j=2,j\neq K}^{K}\sum_{l=2,l\neq j,k} E\left[\rho_{j1}\rho_{jl}\rho_{l1}\rho_{jl}\sqrt{P_j}\sqrt{P_l}\cos(\phi_j-\phi_1)\cos(\phi_l-\phi_j)\cos(\phi_l-\phi_1)\right.\end{array}\right. \\
&= \frac{1}{4N^2}\sum_{k=2}^{K}\sum_{j=1,j\neq k}^{K} P_j
\end{aligned}
$$

The cross-correlation between C and D is,

$$
E[CD] = \frac{(K-3)}{16N^3}\sum_{k=2}^{K}\sum_{j=2,j\neq k} P_j \qquad \text{(A.9)}
$$

The variance of total noise term is

$$
\begin{aligned}
var(\chi_i) &= \frac{N_0}{2T} \\
var(\chi_{p,i}) &= \frac{N_0}{2T}\frac{(K-1)}{2N} \qquad \text{(A.10)}
\end{aligned}
$$

Most of the cross-correlations are zero except the cross-correlation between $I^{(1)}$ and I_{uw}, and can be calculated and is,

$$
E\left[I^{(1)}, I^{(1)}_{uw,i}\right] = \sum_{k=2}^{K}\sum_{j=2}^{K} P_j \qquad \text{(A.11)}
$$

Thus the total interference variance can be evaluated from the above equations.

Appendix B

Acronyms

AWGN Additive White Gaussian Noise

BER Bit Error Rate

BPSK Binary Phase Shift Keying

BSC Binary Symmetric Channel

CDMA Code Division Multiple Access

DS Direct Sequence

DSP Digital Signal Processing

EKF Extended Kalman Filter

FDMA Frequency Division Multiple Access

FH Frequency Hopping

IC Interference Cancellation

IS-95 Interim Standard 95

ISI Inter-Symbol Interference

MAI Multiple Access Interference

ML Maximum Likelihood

MMSE Minimum Mean Square Error

MPIC Modified Parallel Interference Cancellation

MC Multi-Code

PIC Parallel Interference Cancellation

PN Pseudo Noise

QoS Quality of Service

SNR Signal to Noise Ratio

TDMA Time Division Multiple Access

WLS Weighted Least Squares