The Information Content of Multiple Receive Aperture SAR Systems

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Abstract-For SAR to perform correctly, the number of unique measurements obtained by the radar (i.e., the rank of the received signal's covariance matrix) must be greater than the number of pixels illuminated. For a single aperture SAR, the coherent processing interval (CPI) and bandwidth determine the number of independent measurements collected; therefore, the received time-bandwidth product limits the maximum unambiguous illumination area, or swathwidth.

For a multiple aperture SAR (MSAR), however, the rank of the received signal is not as easy to determine. When the array is large, its beamwidth determines resolution rather than the radar's bandwidth and CPI length. Furthermore, redundant lags in the space-time-frequency co-array reduce the amount of unique information collected.

This paper generalizes the theory behind determining the rank of a signal received from stationary targets. Resolution is determined by all radar parameters including CPI length, bandwidth, and array extent. The co-array concept for antenna arrays, which is a measure of the lags sampled in the array's spatial covariance matrix, is extended and applied. A hybrid co-array is derived that indicates lags sampled in the hybrid space-time-frequency space. The hybrid co-array is then applied to signals received by MSAR to show that the number of unique lags in the hybrid co-array limits the number of unique samples collected. The results provide important analysis tools for MSAR systems that are likely in the future, especially sparse, constellation-flying satellite systems.

I. INTRODUCTION

Recently, there has been more interest in multiple aperture SAR [1]. These systems propose multiple flying platforms, each with their own receivers. However, since the receivers are located on independent vehicles that must maintain a safe separation distance, the spacing between elements may be large. Consequently, the arrays for such systems will be large in size and sparsely populated. In addition, if fuel budgets require that satellite platforms orbit with minimal propulsion, the radar designer will not have control over aperture placement. To the radar engineer, the array structure will be known but arbitrary. With large, sparsely populated, arbitrarily sampled arrays, come high sidelobes and the potential for grating lobes. Therefore, we must determine how to evaluate the number of independent samples, resolution, number of illuminated pixels, and ambiguity functions of such systems. The number of illuminated pixels that can be uniquely identified is known as the stationary target rank, or clutter rank for moving target indication (MTI). For unambiguous SAR images, stationary target rank must be equal to the number of illuminated resolution cells. However, it is difficult to determine these system parameters for the types of arrays mentioned. For example, the axes of the resolution ellipse can rotate away from the along-track and cross-track directions when array beamwidth determines resolution rather than bandwidth and CPI length.

In Section II, we derive a rule for stationary target rank for the sidelooking case and show that it reduces to Brennan's rule [2] with the correct assumptions. In Section III, we extend the co-array concept for antenna arrays to a synthetic co-array made from data sampled in spatial position, time, and frequency. The number of unique lags sampled in this co-array is an upper bound to the rank of the received signal. In Section IV, we present a method for determining the synthetic co-array and true axes of the resolution ellipse for non-sidelooking geometries and large arrays. Our conclusions are in Section V.

II. RANK PREDICTION FOR SIDELOOKING GEOMETRIES

The geometry used in this paper is the same as used by [2] and is shown in Fig. 1. The receive platforms all travel in the x-direction. For sidelooking radar, the approximate received signal as a function of frequency, time, and spatial position is

$$s\left(f,t,r_n^x,r_n^y,r_n^z\right)=\exp\left[jk_0\cos\theta\sin\phi\left(r_n^x+2vt\right)\right] \exp\left\{\begin{array}{l} jk_0\sin\left(\theta_i-\theta\right)\bullet \\ \left[r_n^y\sin\theta_i+r_n^z\cos\theta_i-\dfrac{2hf}{f_0\sin\theta_i\tan\theta_i}\right]\end{array}\right\}, \qquad (1)$$

where k_0 is the wavenumber at the carrier frequency, f_0 is the carrier frequency, $r_n^{x,y,z}$ are the x, y, and z locations of the nth receiver, v is the velocity of the radar platform, θ_i is the elevation angle to the target at the center of illumination, h is the altitude at the center of the array, and f is the baseband frequency of the received pulse. The narrowband

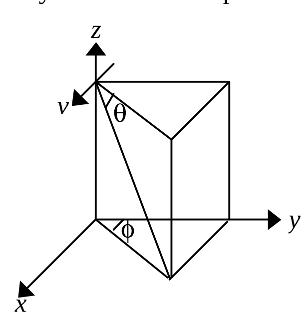

Fig. 1. SAR geometry.

approximation for the spatial component has been applied. Also, a narrow beamwidth has been assumed so that range can be approximated using the small angle sine approximation. In (1), we see that the along-track component is a sinusoid of spatial frequency $\cos\theta\sin\phi/\lambda_0$ sampled at a position given by ($r_n^x + 2vt$). Van Trees [3] said, "…when a bandlimited process [-W, W] is observed over a T-second interval, there are only (2TW + 1) significant eigenvalues." Hence, the rank of the along-track signal is approximately

$$r_{az} = \frac{1}{\lambda_0}\cos\theta_i\left[\sin\phi^{\max} - \sin\phi^{\min}\right]\left[L_x + 2vT\right] + 1\,, \qquad (2)$$

where L_x is the array's along-track size and T is the CPI length. Writing (2) in terms of along-track spatial bandwidth, B_s^{az}, and Doppler bandwidth B_D, the stationary target rank is

$$r_{az} = B_s^{az} L_x + B_D T + 1\,. \qquad (3)$$

With a similar approach, the cross-track component's rank is

$$r_{el} = B_s^{el} L_{el} + B\Upsilon + 1\,, \qquad (4)$$

where B_s^{el} is the elevation spatial bandwidth, L_{el} is the cross-track array extent orthogonal to the vector directed toward the center of the illuminated area, B is the system bandwidth, and Υ is the width of illuminated delays. Since the along-track and cross-track components are independent, the total rank observed due to a 2-D patch of ground is the product of the rank of the two components,

$$r_s = r_{az} r_{el}\,. \qquad (5)$$

With the proper assumptions, (5) reduces to Brennan's rule given in [2]. First, rank calculations are for a single range bins so that r_{el} = 1. Second, an airborne, sidelooking scenario is assumed so that θ_i is small. Third, we assume a fully filled array with half-wavelength interelement spacing, d, and pulse repetition interval (PRI) related to the spacing by,

$$PRI = {\beta d}/{2v} = {\beta\lambda_0}/{4v}\,. \qquad (6)$$

At this point, the rank is

$$r_s = r_{az} = B_s^{az}\left(N-1\right)\lambda_0/2 + B_D\left(M-1\right)\mathrm{PRI} + 1\,, \qquad (7)$$

where N is the number of receive elements and M is the number of transmitted pulses. Last, if the antennas have no directivity in azimuth, then B_s^{az} is $2/\lambda_0$, B_D is $4v/\lambda_0$, and using (6) we arrive at Brennan's rule,

$$r_s = N + \left(M-1\right)\beta\,. \qquad (8)$$

III. Synthetic Array and Co-array

In reality, the above rules for the rank of illuminated stationary targets are bounds, not always equalities. There must also be enough unique *lags* in the space-time-frequency measurement covariance matrix. Referring to array theory, we see that a sparse array can still have an unambiguous pattern if the co-array is properly filled [5]. The co-array is the autocorrelation of the physical array. Applying this theory to our situation, we define a synthetic array out of our measurements in space, time, and frequency, and then take the autocorrelation of the synthetic array to get the synthetic co-array. The number of unique lags sampled in the synthetic co-array bounds the stationary target rank.

First, we define a hybrid coordinate system that transforms space, time, and frequency samples into a 2-D synthetic array. For sidelooking, one dimension is obvious and corresponds to the traditional synthetic array that is the basis for SAR. This dimension is in the along-track direction with samples located according to

$$\tilde{x} = r_n^x + 2vt_m \qquad (9)$$

for every receiver and transmitted pulse. The second dimension is perpendicular to the along-track direction and to the position vector pointing from the center of the array at time zero to the center of the illuminated area with samples located at

$$\tilde{y} = r_n^y \sin\theta_i + r_n^z \cos\theta_i - \frac{2hf_l}{f_0 \sin\theta_i \tan\theta_i}\,. \qquad (10)$$

Note that (9) and (10) are seen in (1) where they sample spatial frequencies proportional to $\cos\theta\sin\phi$ and $\sin\left(\theta_i - \theta\right)$, respectively.

Using (9) and (10), we form a 2-D synthetic array from our measurements in space, time, and frequency, then correlate the array with itself to get a co-array. The magnitude of the co-array indicates the number of times a particular lag is sampled while the number of nonzero elements of the co-array indicates the number of unique lags sampled. This gives a tool for determining if a particular multiple aperture system satisfies the sampling requirements necessary to unambiguously identify all illuminated pixels. A sample synthetic array and it co-array are shown in Fig. 2.

IV. Extension to All Look Geometries and Arrays of Large Extent

Thus far only sidelooking geometries have been considered. Also, it has been quietly assumed that resolution is governed by signal bandwidth and CPI length rather than by array size. If either of these assumptions are not valid, then the two independent directions of resolution and the coordinate system of the synthetic array are not easy to determine. We need a method for determining the natural coordinate systems for any given system geometry, including sparse, irregularly sampled arrays.

We write the received phase as a function of all sensor parameters and target position and expand the phase around sensor parameters and target position using Taylor's theorem. Then holding one target position fixed, we allow a second target's position to vary. The resolution ellipse is defined by correlating the phase responses over all sensor parameters and setting the correlation equal to a constant. The result is

$$C^2 = \mathbf{\Delta x}'\mathbf{\Lambda}_0' \int_S \mathbf{\Delta s}\mathbf{\Delta s}' \mathrm{dS}\, \mathbf{\Lambda}_0 \mathbf{\Delta x} = \mathbf{\Delta x}'\mathbf{\Lambda}_0' \mathbf{J}_s \mathbf{\Lambda}_0 \mathbf{\Delta x}\,, \quad (11)$$

where $(\ )'$ denotes the transpose operation, $\mathbf{\Delta x}$ is the vector of position deviation from the first target, $\mathbf{\Delta s}$ is the deviation of sensor parameters from their mean values, $\mathbf{\Lambda}_0$ is a matrix defined as

$$\mathbf{\Lambda}_0 = \nabla_s \left[\nabla_x \Psi(\mathbf{s}, \mathbf{x}) \right]' \Big|_{\bar{\mathbf{s}}, \bar{\mathbf{x}}}\,, \quad (12)$$

∇_s is the gradient operator over sensor parameters, ∇_x is the gradient operator over target position, $\Psi(\mathbf{s}, \mathbf{x})$ is the phase function, $\bar{\mathbf{s}}$ is the vector of mean sensor parameters, $\bar{\mathbf{x}}$ is the position vector of the first target, and the integration is performed over the range of all sensor parameters. Equation (11) defines an ellipse, and the axes of the ellipse are the eigenvectors of $\mathbf{\Lambda}_0' \mathbf{J}_s \mathbf{\Lambda}_0$. Also, if we compute the singular value decomposition of $\mathbf{\Lambda}_0$, the two basis vectors for its columns project our 5-D measurement vectors into a 2-D synthetic array. Again, the autocorrelation of the synthetic array produces the synthetic co-array that can be used to determine how many unique lags have been sampled. The two basis vectors for the rows of $\mathbf{\Lambda}_0$ project the (x,y) coordinates of the targets into the two spatial directions measured by the axes of the synthetic array. The ambiguity function for an arbitrary geometry and array has been calculated and is shown in Fig. 3. The ellipse calculated using (11) is plotted over the top of the ambiguity function and shows excellent agreement.

V. Conclusion

We have proposed a method for calculating the axes of the resolution ellipse for a multiple aperture system of arbitrary size and sample distribution. Once two independent directions of resolution are found, they can be used to predict the number of pixels, or stationary target rank, illuminated by a radar system. We have shown that the total rank is the product of the rank of two independent dimensions.

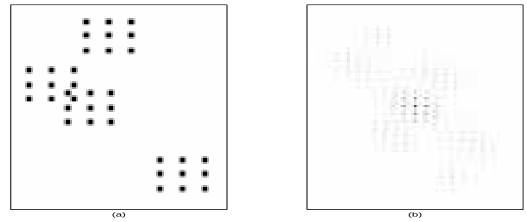

Fig. 2. (a) Synthetic array and (b) co-array made from 4 receivers with 3 frequency samples and 3 time samples each.

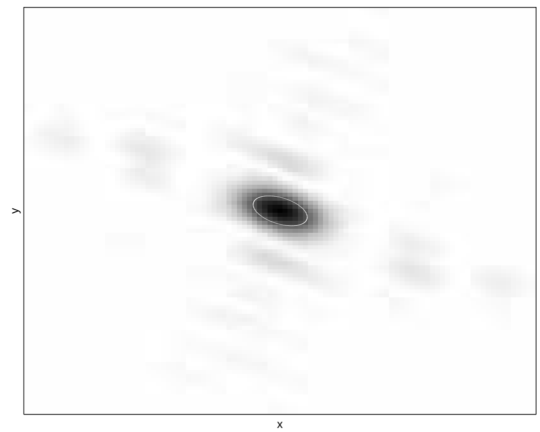

Fig. 3. Ambiguity function for a forward-looking scenario.

Furthermore, we have established a foundation for calculating the rank in each dimension, rather than relying on intuitive arguments and strict assumptions as has been done in the past for traditional MTI systems. When the proper assumptions are made, we agree with previous expressions for rank. Last, we have described how to compute the synthetic co-array from space, time, and frequency data and described how redundant lags may result in reduced rank and ambiguities.

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