Performance Analysis of a DS-CDMA Packet Cellular/PCS Network in a Multi-Rate Multi-User Environment

Manish Mangal[1] and James A. Roberts
Information and Telecommunications Technology Center
Department of Electrical Engineering and Computer Science
The University of Kansas, Lawrence, KS 66045 USA

Douglas. L. Machamer
Technology Planning and Integration Group
Long Distance Division, Sprint Corporation
Overland Park, KS 66212 USA

Abstract

The delay, throughput and capacity analysis of a DS-CDMA packet cellular network is carried out in this paper. A mix of data and voice user classes is assumed. Error free communication is required for the data users but is not delay sensitive while voice applications allow some degree of error but is delay sensitive. The QoS for data users is therefore the delay while for voice it is the bit error rate (BER).

1. System Model

The system model for an integrated data and voice DS-CDMA packet cellular network is shown in Fig. 1. A stop-and-wait (SW) protocol is considered for data users, in which errored packets are discarded by the receiver and are requested to retransmit, thereby providing error free communication for the data users but incurring a delay due to retransmissions. Since voice is delay sensitive, the voice packets are not retransmitted and hence the QoS for the voice users is the bit error rate, while for the data users delay is the QoS measure. In Fig. 1, K_d represents the number of data packets in the system. K_{re} is the number of retransmitted data packets and depends on the probability of packet error which in turn is a function of the total number of packets in the wireless channel. K_n is the number of new data packets entering in the wireless channel and K_r is the number of packets correctly received. Increasing K_n will increase the number of packets in the wireless channel which will decrease the probability of packet success for the desired user due to an increase in multiple access interference (MAI), the interference caused to the desired user by the other users. The decrease in the probability of packet success will then increase K_{re} which in turn will increase K_d thereby making the system unstable. The system will be in equilibrium if and only if K_n is equal to K_r.

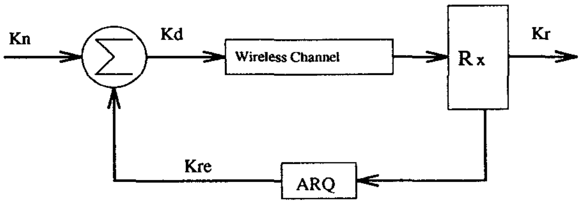

Figure 1. System Model

2. Performance Analysis

The total noise power spectral density in a DS-CDMA cellular system is given by[1],

$$N_o' = N_o + \frac{PR}{\alpha B}(M-1) \tag{1}$$

where B is the available bandwidth and N_o is the power spectral density (PSD) of the background noise. P is the signal power and α is the power control accuracy factor which is 1 in a perfect power control case and is 0.85 [2] in an urban cellular environment. M is the number of users in the system. Including the effect of a pulse shaping factor, sectorization factor, activity factor and other cell interference, the bit energy to total noise density can be expressed as,

$$\frac{E_b}{N_o'} = \frac{1}{\frac{1}{E_b/N_o} + \frac{p\beta}{\alpha n G_p}[(1+\lambda_i)K - 1]} \le \gamma \tag{2}$$

where λ_i is the other cell interference which accounts for the amount of interference received from the users outside the desired user's cell and is taken as 0.5 [3]. n is the sectorization, which refers to subdividing the area around the cell into smaller sections and using directional antennas at the cell site both for transmitting and receiving. G_p is the processing gain defined as the ratio of the bandwidth to the information bandwidth which for QPSK can be taken

---

This work was sponsored by a grant from Sprint Corporation, Overland Park, Kansas

[1] Currently working for Technology Planning and Integration Group, Sprint Corporation, Overland Park, Kansas

0-7803-4320-4/98/$5.00 © 1998 IEEE

as the bit rate. Studies show that either speaker is active only for about 35% to 40% of the time of a typical conversation [4] and is accounted as an activity factor represented by p in (1). It is taken as 3/8 [5, 3] for voice users and varies from 1 to 1/10 for data users depending upon the application. β is the pulse shaping factor and is 1 to 2/3 in extreme cases i.e. band-limited and time-limited respectively. In our analysis we assumed a square root raised cosine pulse shaping filter with a roll-off factor of 0.2 [6]. The pulse shaping factor is therefore 0.7018 [7]. γ is the required bit energy to the total noise density (MAI and thermal noise) to achieve required BER and delay for the voice and data users respectively. E_b/N_o is a bit energy to the thermal noise ratio. Inverting the above expression and using an assumption that half of the allowable noise is contributed by the thermal noise and half is user interference noise [1] i.e. $E_b/N_o = 2\gamma$ in (1), the capacity of a DS-CDMA cellular system is given by

$$K = \frac{1}{1+\lambda_i}\left[1 + \frac{\alpha n G_p}{p\beta}\frac{1}{2\gamma}\right] \tag{3}$$

For QPSK modulation in the channel, the bit error rate is given as

$$\epsilon(\gamma) = \frac{1}{2} erfc(\sqrt{\gamma}) \tag{4}$$

and the packet error rate (P_{pe}) assuming ideal interleaving is given as

$$P_{pe} = 1 - (1 - \epsilon(\gamma))^N \tag{5}$$

where N is the packet length. Substituting (3) and (4) in (5), we get,

$$P_{pe} = 1 - \left\{1 - \frac{1}{2} erfc\left(\sqrt{\frac{1}{\frac{2p\beta}{\alpha n G_p}[(1+\lambda_i)K - 1]}}\right)\right\}^N \tag{6}$$

It can be seen from Fig. 2 that the probability of packet success (P_{ps}), i.e. $1 - P_{pe}$, increases with a decrease in the packet length (N) and also shows that better power control improves the P_{ps} as expected.

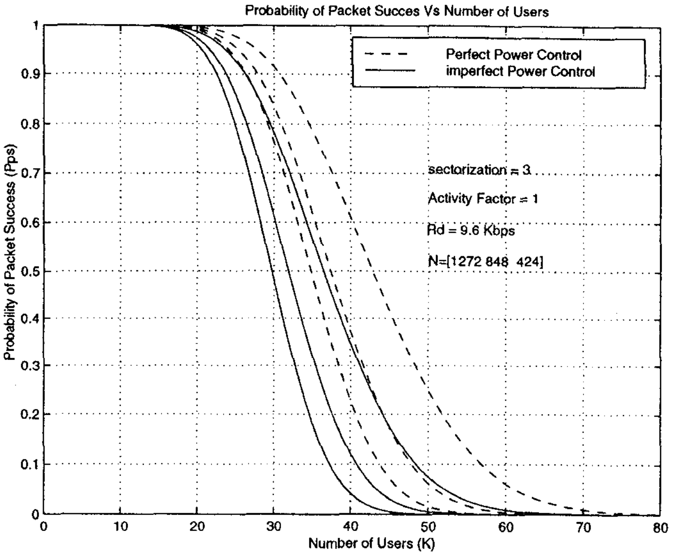

Figure 2. Probability of Data Packet Success Vs. Number of Data Users

3. Single class of data users

In this section, we find the average channel delay and the throughput for a single class of data users as a function of the total number of data users. The capacity equation (3) is then modified for the data users to be able to express the capacity as the function of the QoS, i.e. *delay*. The system model is shown in Fig. 1.

3.1. Delay

Neglecting the effect of queueing, the average channel delay (D) for the data packets is given by,

$$D = \frac{D_{min}}{1 - P_{pe}} \tag{7}$$

where P_{pe} is the probability of a packet error and is the same as the probability of a retransmission, and D_{min} is the minimum transmission time of a packet and comprises packetizing, propagation and processing delay. The propagation and processing delay (t_d) is very low and is assumed to be 3 ms [8] in our analysis, while the packetization delay is given by the ratio of packet length to the information data rate. Thus D_{min} is given by,

$$D_{min} = \frac{N}{R_d} + t_d \tag{8}$$

and the average delay for the data packets is given by

$$D = \frac{D_{min}}{\left\{1 - \frac{1}{2} erfc\left(\sqrt{\frac{1}{\frac{2p\beta}{\alpha n G_p}[(1+\lambda_i)K_d - 1]}}\right)\right\}^N}. \tag{9}$$

Fig. 3 shows that there is almost no increase in the delay for the data packets initially with the increase in the number of users because of a fairly constant P_{ps} for a low number of users as seen in Fig. 2, but a point is reached after which the P_{ps} starts falling down rapidly and thus requiring a higher number of retransmissions. Beyond that, a slight increase in the number of users causes a high increase in the delay for data packets. So the operating region on this plot would be the knee area of the plot to obtain the best compromise for the capacity and the delay. The increase in the packet length causes an increase in the delay because of the increase in the packetizing delay and the decrease in the probability of packet success.

3.2. Throughput

Throughput (S) here is defined as the total number of packets that can be correctly received per second at the

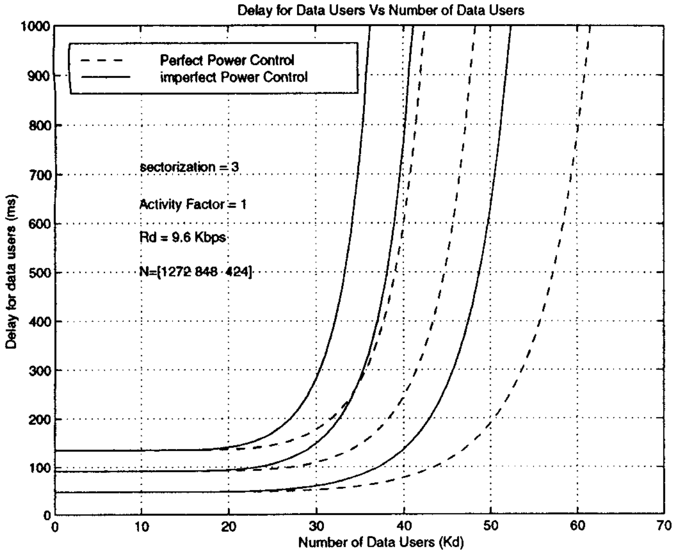

Figure 3. Delay for Data Users Vs. Number of Data Users

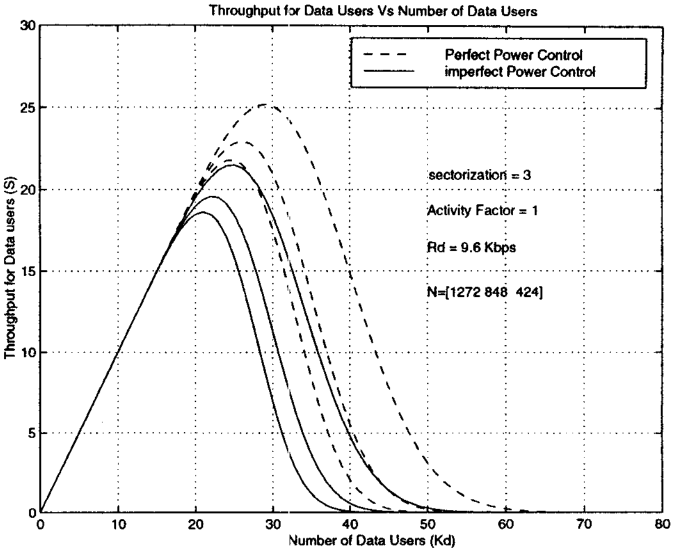

Figure 4. Throughput Vs. Number of Users

receiver and is a function of total number of packets in the wireless channel. From Fig. 1, we have,

$$K_n = K_d - K_d P_{pe} = K_d P_{ps} \tag{10}$$

which gives the number of new packets that can enter on the channel to maintain the equilibrium of the system. Thus the throughput can be written as,

$$S = K_n \frac{R_d}{N} = K_d P_{ps} \frac{R_d}{N} \tag{11}$$

Substituting (6) in (11), we have,

$$S = K_d \frac{R_d}{N} \left\{ 1 - \frac{1}{2} erfc \left\{ \sqrt{\frac{\alpha n G p}{2p\beta[(1+\lambda_i)K_d - 1]}} \right\} \right\}^N \tag{12}$$

It can be seen from Fig. 4 that the throughput increases with the number of users till a certain point, this point corresponds to the bending point on Fig. 3. After that, the number of packets correctly received starts decreasing because of rapid decrease in the P_{ps}, although more network resources i.e PN codes are used. The operational point on this curve should be the peak of the curve.

3.3. Capacity

The capacity equation (3) can be modified for the data users to be able to express it as a function of the QoS, i.e. *delay* for the data users and the BER for the voice users. Using (4), (5) and (7), the required γ for the data users can be written as a function of *delay*, as,

$$\gamma_d = \left\{ erf^{-1} \left\{ 1 - 2 \left[1 - \left(\frac{D_{min}}{D} \right)^{1/N} \right] \right\} \right\}^2 \tag{13}$$

and for the voice users, γ can be written as a function of packet error rate (BER), as,

$$\gamma_v = \left\{ erf^{-1}(1 - 2BER) \right\}^2 \tag{14}$$

Using (15) in (3), the capacity for data users as a function of delay when there are no voice users present, can be written as,

$$K_d = \frac{1}{1+\lambda_i} \left[1 + \frac{\alpha n G_p}{p\beta} \frac{1}{2\left\{ erf^{-1} \left\{ 1 - 2 \left[1 - \left(\frac{D_{min}}{D} \right)^{1/N} \right] \right\} \right\}^2} \right] \tag{15}$$

Increasing the delay requirements means allowing more retransmissions, i.e. a higher probability of a packet error, and this in turn means allowing more users on the wireless channel thereby achieving an increase in the capacity at the cost of the QoS.

4. J Classes of Users

In this section the effect on the delay and the throughput for data users due to the integration of voice users is analyzed. The average delay and the throughput equations are generalized for J classes of users. The BER for the voice users is assumed to be 10^{-3}. Equation (2) can be extended for J classes of users and the received bit energy to the total noise density (γ_j) for the j^{th} class of users can be written as,

$$\gamma_j = \frac{1}{\frac{2p_1\beta}{\alpha n G_{p_1}}(1+\lambda_i)K_1 \cdots + \frac{2p_j\beta}{\alpha n G_{p_j}}[(1+\lambda_i)K_j - 1] \cdots + \frac{2p_J\beta}{\alpha n G_{p_J}}(1+\lambda_i)K_J} \tag{16}$$

0-7803-4320-4/98/$5.00 © 1998 IEEE

where, p_j, G_{p_j}, K_j are the activity factor, processing gain and number of jth class of users respectively. Extending (6) for J classes of users, the probability of packet success for the j_{th} class of users is given by,

$$P_{ps_j} = \left\{ 1 - \frac{1}{2} erfc \left(\sqrt{ \frac{1}{\left(\frac{2p_1\beta}{\alpha n G_{p_1}}(1+\lambda_i)K_1 + \frac{2p_j\beta}{\alpha n G_{p_j}}[(1+\lambda_i)K_j - 1] + \cdots \frac{2p_J\beta}{\alpha n G_{p_J}}(1+\lambda_i)K_J \right)} } \right) \right\}^N \quad (17)$$

The capacity is bounded by the following J-dimensional hyperplane [1],

$$\sum_{j=1}^{J} \frac{K_j}{\widetilde{K_j}} \leq 1 \quad (18)$$

where $\widetilde{K_j}$ is the number of j^{th} class of users when no other class of users are present. The delay for the j_{th} class of users can be found as a function of the number of all other classes of users by substituting (17) in (7), and is given by,

$$D_j = \frac{D_{min}}{\left\{ 1 - \frac{1}{2} erfc \left(\sqrt{ \frac{1}{\left(\frac{2p_1\beta}{\alpha n G_{p_1}}(1+\lambda_i)K_1 + \frac{2p_j\beta}{\alpha n G_{p_j}}[(1+\lambda_i)K_j - 1] + \cdots \frac{2p_J\beta}{\alpha n G_{p_J}}(1+\lambda_i)K_J \right)} } \right) \right\}^N} \quad (19)$$

The throughput for the j_{th} class of users can be written by substituting (17) in (11) as,

$$S_j = K_j \frac{R_j}{N} \left\{ 1 - \frac{1}{2} erfc \left(\sqrt{ \frac{1}{\left(\frac{2p_1\beta}{\alpha n G_{p_1}}(1+\lambda_i)K_1 + \frac{2p_j\beta}{\alpha n G_{p_j}}[(1+\lambda_i)K_j - 1] + \cdots \frac{2p_J\beta}{\alpha n G_{p_J}}(1+\lambda_i)K_J \right)} } \right) \right\}^N \quad (20)$$

Let us consider an example of two classes of users, one class being voice users and the other being data users. Fig. 5 shows the probability of data packet success for various combinations of data and voice users. The information rate for voice and data is assumed to be 9.6 Kbps and 14.4 Kbps respectively. The activity factors are taken to be 1 and 3/8 for data and voice users respectively. The chip rate is assumed to be 1.2288 Mcps [6] and the sectorization factor is assumed to be 1. It can be seen that the addition of voice users causes the data packet success probability to drop down because of the increase in the MAI due to the addition of the signal power of voice users.

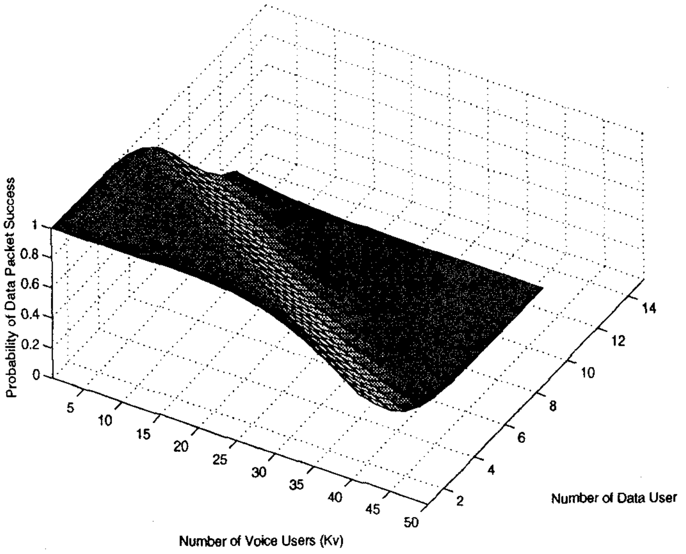

Figure 5. Probability of Data Packet Success Vs. Integrated Voice and Data Users

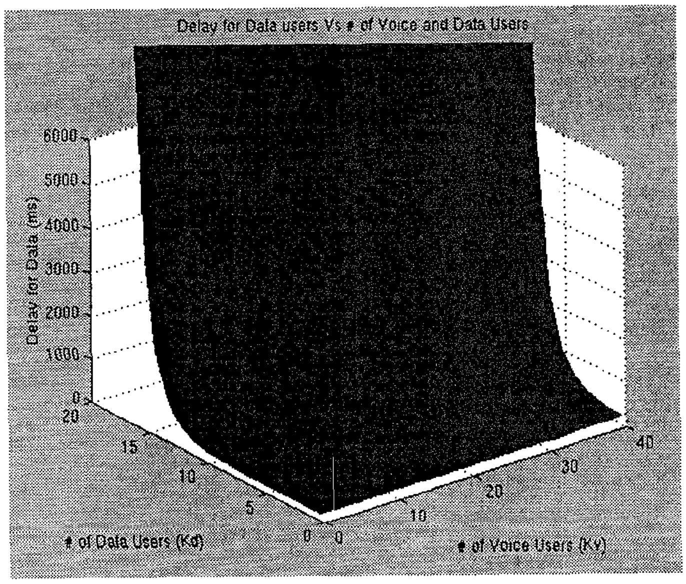

Figure 6. Delay for Data Users Vs. Integrated Voice and Data Users

Fig. 6 shows the effect on the average delay for data users due to addition of voice users. It can be seen from Fig. 6 that when there are no voice users, i.e. $K_v = 0$, the knee area on the curve starts at around $K_d = 35$ and reduces with the addition of voice users and occurs at around $K_d = 7$ when $K_v = 122$. This is because, the addition of voice users causes a decrease in the data packet success probability thereby requiring more retransmissions. Fig. 7 shows the throughput of data packets in an integrated voice and data system. The throughput is maximum when there are no voice users. As the voice users start adding up, the number of data users goes down as per (19) to satisfy the QoS requirements for both voice and data, thereby decreasing the maximum possible data throughput.

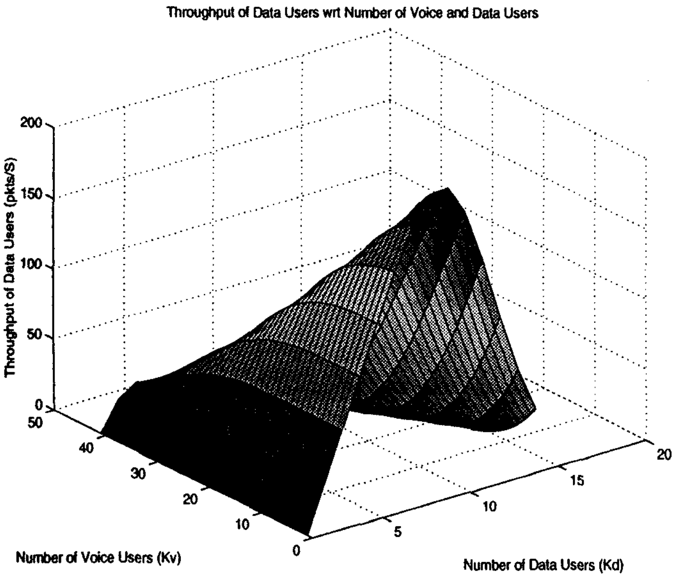

Figure 7. Throughput of Data Users Vs. Integrated Voice and Data Users

Now consider an example for three classes of users. Assume one type of user to be voice with activity factor p of 3/8, BER requirement of 10^{-3} and information bit rate of 9.6 Kbps. The other two classes of users are data, and p and D are assumed to be 0.5 and 500 ms for both of them. The information bit rates for data type 1 and type 2 are assumed to be 14.4 Kbps and 28.8 Kbps respectively. The capacity plot is plotted for this case in Fig. 8. The end points on the curve corresponds to the maximum number of voice, data type 1 and data type 2 users possible in the system, 47, 79 and 42 respectively for this example.

5. Summary

An approach for determining delay and throughput for data users utilizing packet transmissions has been developed. The effect on delay and throughput for data users with the addition of voice users was then studied. The equations were then generalized for J classes of users and the J-dimensional hyperplane was plotted showing the bounds on the capacity of all classes of users where each class is specified by a data rate, an activity factor and a QoS which is delay and packet error rate for data and voice users respectively.

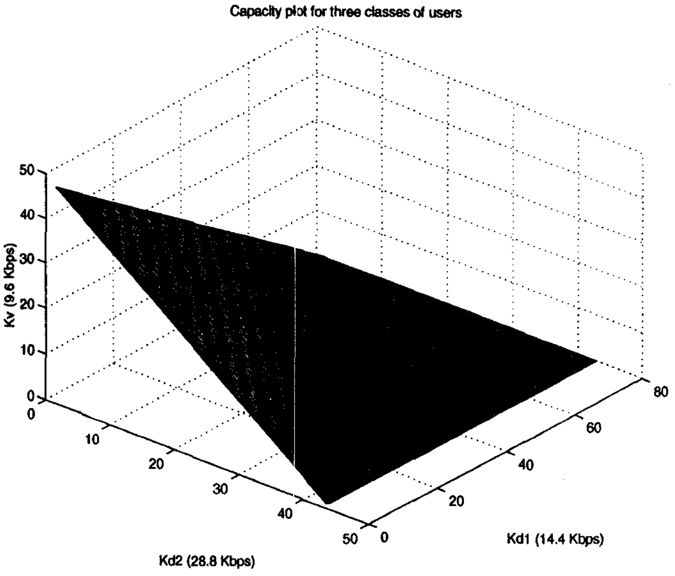

Figure 8. Three classes of users (One voice and two data type)

References

[1] M. Mangal, V. Paulrajan, J. A. Roberts, *CDMA Cellular Capacity Assessment*, Technical Report TISL-11880-01., Dec. 1996.

[2] Vijay K. Garg, E. L. Sneed, "Digital Wireless Local Loop Systems," *IEEE Communications Magazine.*, pp. 112-115, Oct. 1996.

[3] V. K. Paulrajan, James A. Roberts, and D. L. Machamer, "Capacity of a CDMA Cellular System with Variable User Data Rates," *Proc. IEEE Globecom*, London, UK, Nov. 96, vol. 3, pp. 1458-1462.

[4] P. T. Brady, "A statistical analysis of on-off patterns in 16 conversations," *Bell Syst. Tech. J.*, vol. 47, pp. 73-91, Jan. 1968.

[5] K.S. Gilhousen, I.M. Jacobs, et al, "On the Capacity of a Cellular CDMA System," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 303-312, May 1991.

[6] TIA/EIA IS-95, "Mobile Station-Base Station Compatibility Standard for Dual-Mode Wideband Spread Spectrum Cellular System."

[7] Manish Mangal, *Capacity, Delay and Throughput Analysis of a DS-CDMA Packet Cellular Network in a Multi-Rate Multi-User Environment*, M.S. Thesis, The University of Kansas, Lawrence, KS, July 1997 .

[8] M. L. Honig, J. B. Kim, "Resource Allocation for Packet Data Transmission in DS-CDMA," *Proc. Allerton Conference 95*, pp. 925-934, Urbana-Champaign, IL, October 1995.